Al-Huda Science Academy

Mathematics Notes

Class 12
Exercise 1.1

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Exercise 1.1

Q1:- Given that

a) $f(x) = x^2 - x$ b) $f(x) = \sqrt{x+4}$

Find i) $f(-2)$ ii) $f(0)$ iii) $f(x-1)$

iv) $f(x^2+4)$

Answer

a) $f(x) = x^2 - x$ —— ①

i) $f(-2)$

Put $x = -2$ in eq ①

$f(-2) = (-2)^2 - (-2) = 4 + 2 = 6$

ii) $f(0)$

Put $x = 0$ in eq ①

$f(0) = (0)^2 - 0 = 0 - 0 = 0$

iii) $f(x-1)$

Put $x = x-1$ in eq ①

$f(x-1) = (x-1)^2 - (x-1) = x^2 - 2x + 1 - x + 1$

$= x^2 - 3x + 2$

iv) $f(x^2+4)$

Put $x = x^2+4$ in eq ①

$f(x^2+4) = (x^2+4)^2 - (x^2+4) = x^4 + 8x^2 + 16 - x^2 - 4$

$= x^4 + 7x^2 + 12$

b) $f(x) = \sqrt{x+4}$ —— ②

i) $f(-2)$

Put $x = -2$ in eq ②

$f(-2) = \sqrt{-2+4} = \sqrt{2}$

ii) $f(0)$

Put $x = 0$ in eq ②

$f(0) = \sqrt{0+4} = \sqrt{4} = 2$

iii) $f(x-1)$

Put $x = x-1$ in eq ②

$f(x-1) = \sqrt{x-1+4} = \sqrt{x+3}$

iv) $f(x^2+4)$

Put $x = x^2+4$ in eq ②

$f(x^2+4) = \sqrt{x^2+4+4} = \sqrt{x^2+8}$

Q2:- Find $\frac{f(a+h)-f(a)}{h}$ and

Simplify where,

i) $f(x) = 6x - 9$

Put $x = a+h$

$f(a+h) = 6(a+h) - 9$

Put $x = a$

$f(a) = 6a - 9$

Using,

$$\frac{f(a+h)-f(a)}{h} = \frac{6(a+h)-9-(6a-9)}{h}$$

$$= \frac{\cancel{6a}+6h-\cancel{9}-\cancel{6a}+\cancel{9}}{h}$$

$$= \frac{6\cancel{h}}{\cancel{h}}$$

$$= 6$$

ii) $f(x) = \sin x$

Put $x = a+h$

$f(a+h) = \sin(a+h)$

Put $x = a$

$f(a) = \sin a$

Using,

$$\frac{f(a+h)-f(a)}{h} = \frac{\sin(a+h) - \sin a}{h}$$

Using Formula,

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$= \frac{2\cos\left(\frac{a+h+a}{2}\right)\sin\left(\frac{\cancel{a}+h-\cancel{a}}{2}\right)}{h}$$

$$= \frac{2\cos\left(\frac{2a+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \frac{2}{h}\cos\left(a+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$$

iii) $f(x) = x^3 + 2x^2 - 1$

Put $x = a+h$

$f(a+h) = (a+h)^3 + 2(a+h)^2 - 1$

$= a^3 + h^3 + 3a^2h + 3ah^2 + 2(a^2 + 2ah + h^2) - 1$

$= a^3 + h^3 + 3a^2h + 3ah^2 + 2a^2 + 4ah + 2h^2 - 1$

Put $x = a$

$f(a) = a^3 + 2a^2 - 1$

Using,

$$\frac{f(a+h) - f(a)}{h} = \frac{a^3 + h^3 + 3a^2h + 3ah^2 + 2a^2 + 4ah + 2h^2 - 1 - (a^3 + 2a^2 - 1)}{h}$$

$$= \frac{\cancel{a^3} + h^3 + 3a^2h + 3ah^2 + \cancel{2a^2} + 4ah + 2h^2 \cancel{-1} \cancel{-a^3} \cancel{-2a^2} \cancel{+1}}{h}$$

$$= \frac{h^3 + 3a^2h + 3ah^2 + 4ah + 2h^2}{h}$$

$$= \frac{\cancel{h}(h^2 + 3a^2 + 3ah^2 + 4a + 2h)}{\cancel{h}}$$

$= 3a^2 + 4a + 3ah^2 + 2h + h^2$

$= 3a^2 + 4a + h(3ah + 2) + h^2$

iv) $f(x) = \cos x$

Put $x = a+h$

$f(a+h) = \cos(a+h)$

Put $x = a$

$f(a) = \cos a$

Using,

$$\frac{f(a+h) - f(a)}{h} = \frac{\cos(a+h) - \cos a}{h}$$

Using formula,

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$= \frac{-2\sin\left(\frac{a+h+a}{2}\right)\sin\left(\frac{\cancel{a}+h-\cancel{a}}{2}\right)}{h}$$

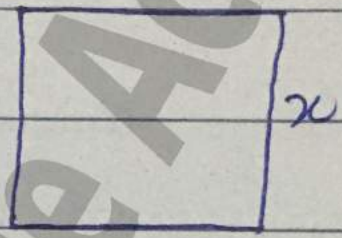
$$= \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \frac{-2}{h}\sin\left(a+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)$$

Q3:- Express the following:

a) The perimeter P of a square as a function of its area A.

Answer



Let 'x' be the side of a square.

Area of Square = side × side

$$A = x.x$$

$$A = x^2 \Rightarrow \sqrt{A} = x$$

Perimeter of Square = 4 × side

$$P = 4x \quad \text{—(1)}$$

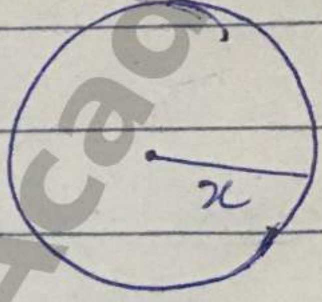
Put $x = \sqrt{A}$ in eq (1)

$$\boxed{P = 4\sqrt{A}}$$

b) The area A of a circle as a function of its circumference.

Answer

Let 'x' be the radius of a circle.



Circumference of circle $= C = 2\pi x$

$$\frac{C}{2\pi} = x$$

Area of circle $= \pi x^2$

$$A = \pi x^2 \quad \text{---} \ (1)$$

Put $x = \dfrac{C}{2\pi}$ in (1)

$$A = \pi \left(\frac{C}{2\pi}\right)^2$$

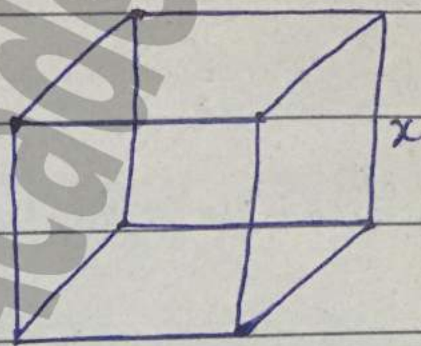
$$A = \cancel{\pi}\frac{C^2}{4\pi^{\cancel{2}}}$$

$$\boxed{A = \frac{1}{4\pi}C^2}$$

c) The volume V of a cube as the function of area A of its base.

<u>Answer</u>

Let 'x' be the side of a cube.



Area of cube = side x side

$$A = x.x$$

$$A = x^2$$

$$\sqrt{A} = x$$

Volume of cube = side x side x side

$$V = x.x.x$$

$$V = x^3 \quad \text{—} \quad (1)$$

Put $x = \sqrt{A}$ in eq (1)

$$V = (\sqrt{A})^3$$

$$V = (A^{\frac{1}{2}})^3$$

$$\boxed{V = A^{\frac{3}{2}}}$$

Q4:- Find the domain and range of function g defined below,

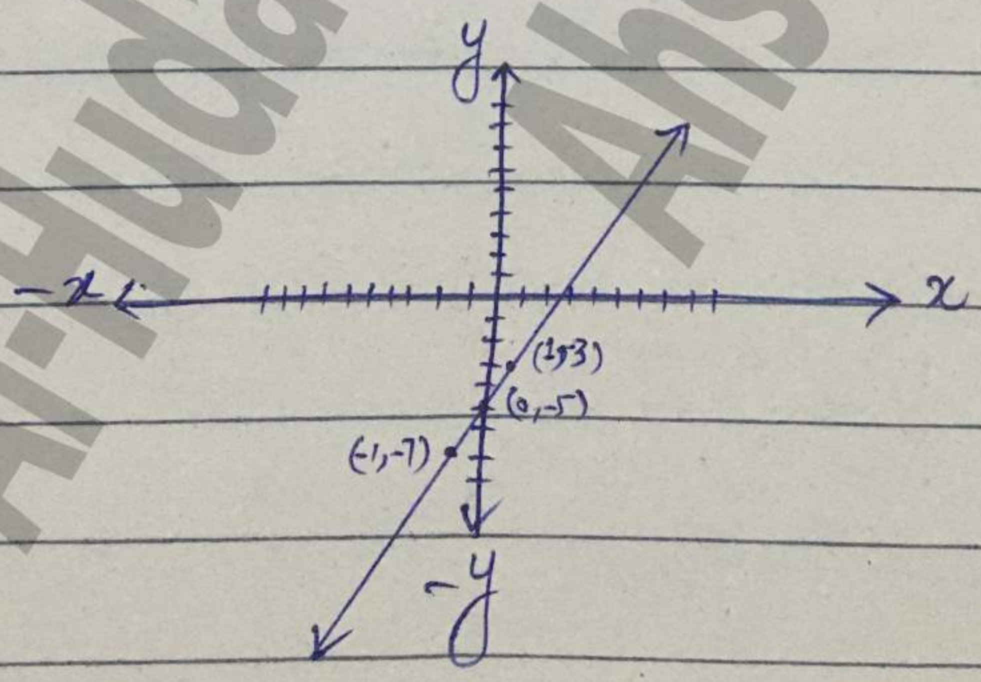
i) $g(x) = 2x-5$

<u>Answer</u>

Domain $g(x)$ = set of all real numbers

Range $g(x)$ = set of all real numbers

x	-1	0	1
$g(x)$	-7	-5	-3



(ii) $g(x) = \sqrt{x^2 - 4}$

<u>Answer</u>

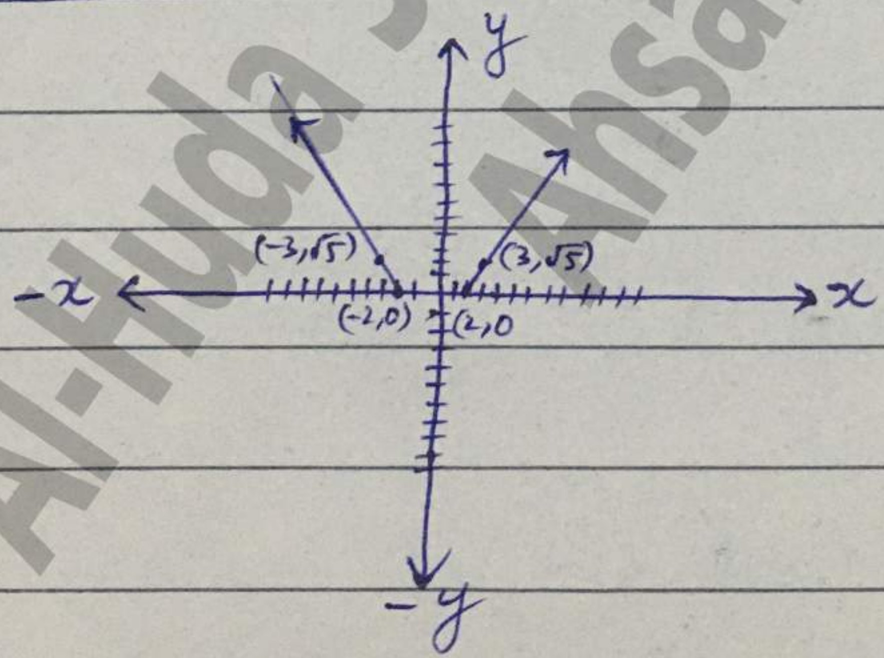
Domain $g(x) = (-\infty, -2] \cup [2, \infty)$

$\because x^2 - 4 \geq 0$ iff $x^2 \geq 4 \Rightarrow x^2 \geq 2^2$ means $x \leq -2$ or $x \geq 2$

Range $g(x) = [0, \infty)$

$\because x^2 - 4 = 0$ if $x = \pm 2$

x	-2	2	3	-3
$g(x)$	0	0	$\sqrt{5} = 2.24$	$\sqrt{5} = 2.24$



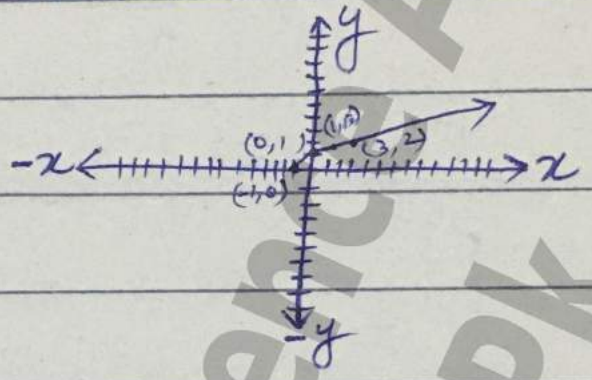
iii) $g(x) = \sqrt{x+1}$

Answer

Domain $g(x) = [-1, \infty)$

Range $g(x) = [0, \infty)$

x	-1	0	1	3
$g(x)$	0	1	$\sqrt{2} = 1.41$	2



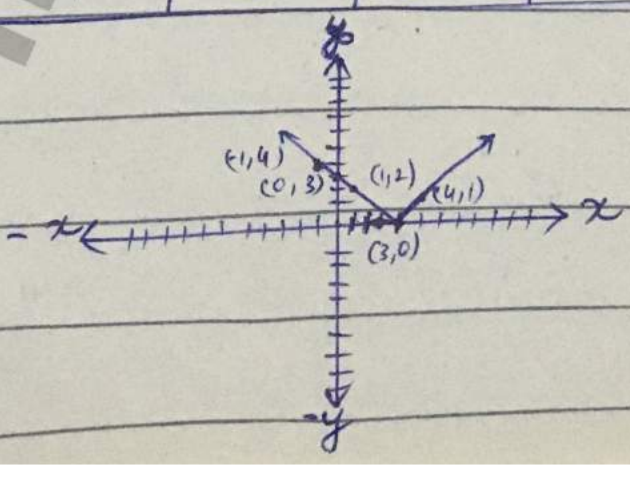
iv) $g(x) = |x-3|$

Answer

Domain $g(x) = (-\infty, \infty)$

Range $g(x) = [0, \infty)$

x	-1	0	1	2	4
$g(x)$	4	3	2	1	1



v) $g(x) = \begin{cases} 6x+7, & x \le -2 \\ 4-3x, & -2 < x \end{cases}$

Answer

Domain $g(x) = (-\infty, \infty)$

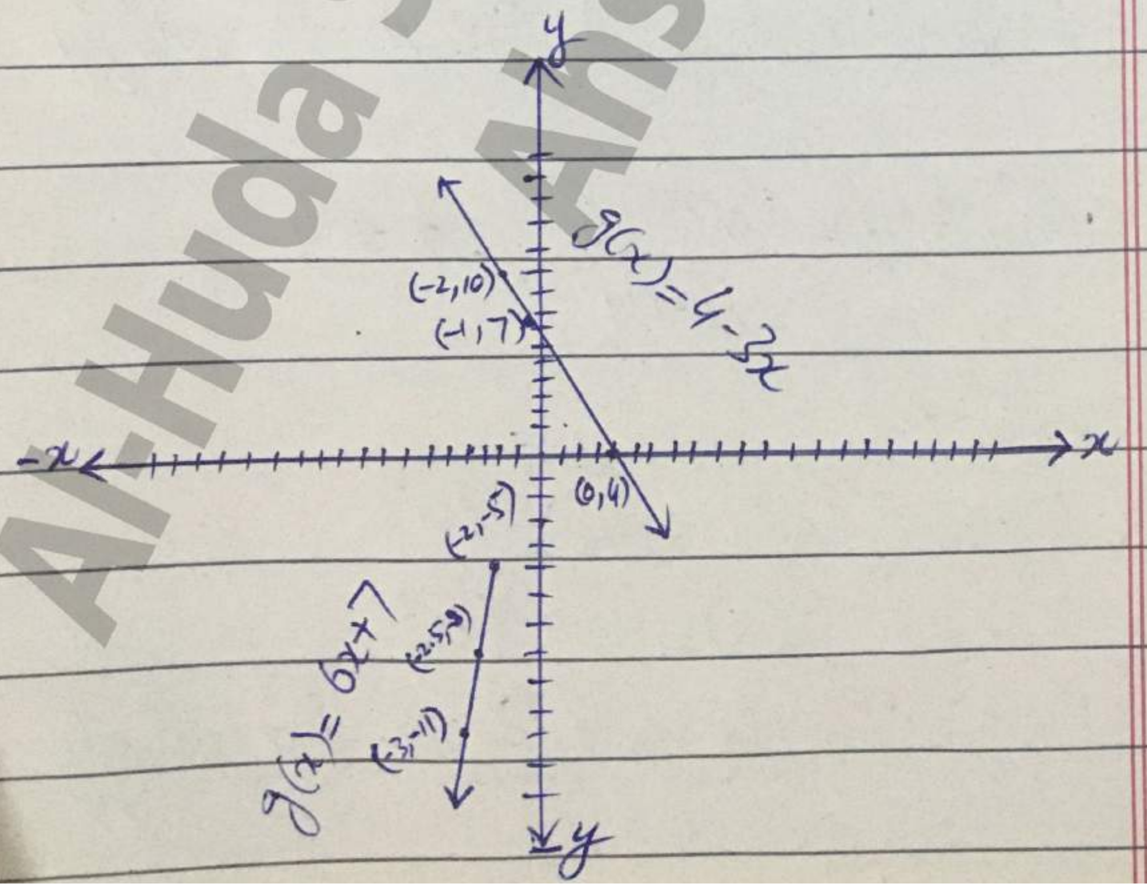
Range $g(x) = (-\infty, 10]$

Table for $g(x) = 6x+7$ when $x \le -2$

x	-2	-2.5	-3
$g(x)$	-5	-8	-11

Table for $g(x) = 4-3x$ when $-2 < x$

x	-2	-1	0		
$g(x)$	10	7	4		



vi) $g(x)\begin{cases} x-1, & x<3 \\ 2x+1, & 3\leq x \end{cases}$

Answer

Domain $g(x) = (-\infty, \infty)$

Range $g(x) = (-\infty, 2) \cup [7, \infty)$

Draw Table and Graphs Accordingly

vii) $g(x) = \begin{cases} \dfrac{x^2-3x+2}{x+1}, & x \neq 1 \end{cases}$

Answer

Domain $g(x) = R - \{-1\}$

Range $g(x) = R - (-5-2\sqrt{6}, -5+2\sqrt{6})$

Draw Table and Graph Accordingly

viii) $g(x) = \dfrac{x^2-16}{x-4}, x \neq 4$

Answer

Domain $g(x) = R - \{4\}$

Range $g(x) = R - \{8\}$

Draw Table and Graph Accordingly

Q.5:- Given $f(x) = x^3 - ax^2 + bx + 1$

If $f(2) = -3$ and $f(-1) = 0$.

Find the values of 'a' and 'b'.

Answer

$$f(x) = x^3 - ax^2 + bx + 1 \quad \text{---} \; (1)$$

Put $x = 2$ in (1)

$$f(2) = 2^3 - a(2)^2 + b(2) + 1$$

$$= 8 - 4a + 2b + 1$$

$$= -4a + 2b + 9$$

As, $f(2) = -3$

Then

$$-4a + 2b + 9 = -3$$

$$-4a + 2b = -3 - 9$$

$$-2(2a - b) = -12$$

$$2a - b = 6 \quad \text{---} \; (2)$$

Put $x = -1$ in (1)

$$f(-1) = (-1)^3 - a(-1)^2 + b(-1) + 1$$

$$= \cancel{-1} - (a) - b + \cancel{1}$$

$$= -a - b$$

As, $f(-1) = 0$

Then

$$a - b = 0 \quad \text{--- } (3)$$

Subtracting (2) & (3)

$$\begin{array}{rl} 2a - \cancel{b} &= 6 \\ -a - \cancel{b} &= 0 \\ + \quad + \quad & \; - \\ \hline 3a \quad &= 6 \end{array}$$

$$a = \frac{6}{3}$$

$$a = 2$$

Put $a = 2$ in (3)

$$2 - b = 0$$

$$\Rightarrow b = -2$$

Hence,

$$a = 2, \; b = -2$$

Q.6:- A stone falls from a height of 60m on the ground, the height 'h' after 'x' second is approximately given by $h(x) = 40 - 10x^2$

i) What is the height of stone

a) $x = 1$ sec

Answer

$$h(x) = 40 - 10x^2$$

Put $x = 1$

$$h(1) = 40 - 10(1)^2 = 40 - 10 = 30$$

b) $x = 1.5$ sec

Answer

$$h(x) = 40 - 10x^2$$

Put $x = 1.5$

$$h(1.5) = 40 - 10(1.5)^2 = 40 - 22.5 = 17.5$$

c) $x = 1.7$ sec

Answer

$$h(x) = 40 - 10x^2$$

Put $x = 1.7$

$$h(1.7) = 40 - (1.7)^2 = 40 - 28.9 = 11.1$$

ii) When does the stone strike the ground?

Answer

Put $h(x) = 0$

$$40 - 10x^2 = 0$$

$$\Rightarrow 10x^2 = 40$$

$$x^2 = 4$$

$$x = 2$$

Hence, the stone strikes the ground after 2 sec.

Q.7- Show that the parametric equations

i) $x = at^2, y = 2at$ represent the equation of parabola $y^2 = 4ax$

Answer

$$x = at^2 \quad — (1) \qquad y = 2at \quad — (2)$$

$$\frac{y}{2a} = t$$

Put $t = \frac{y}{2a}$ in (1)

$$x = a\left(\frac{y}{2a}\right)^2$$

$$x = \cancel{a}\frac{y^2}{4a^{\cancel{2}}}$$

$$4ax = y^2$$

$$\Rightarrow y^2 = 4ax$$

It is equation of parabola.

ii) $x = a\cos\theta,\ y = a\sin\theta$ represent the equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Answer

$x = a\cos\theta \quad , \quad y = b\sin\theta$

$\frac{x}{a} = \cos\theta$ — ①, $\frac{y}{b} = \sin\theta$ ②

Squaring and adding ① & ②

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \because (\cos^2\theta + \sin^2\theta = 1)$$

It is equation of ellipse.

iii) $x = a\sec\theta,\ y = b\tan\theta$ represent the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Answer

$x = a\sec\theta$ — $\quad , \quad y = b\tan\theta$

$\frac{x}{a} = \sec\theta$ — ① , $\frac{y}{b} = \tan\theta$ — ②

Subtracting and Squaring ① & ②

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

It is equation of hyperbola.

Q8:-

Formulas Used

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

For Reciprocals

$$\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

i) $\sinh 2x = 2\sinh x \cosh x$

$$\frac{e^{2x}-e^{-2x}}{2} = \cancel{2} \times \frac{e^{x}-e^{-x}}{\cancel{2}} \times \frac{e^{x}+e^{-x}}{2}$$

$$'' = \frac{(e^{x})^2 - (e^{-x})^2}{2}$$

$$'' = \frac{e^{2x}-e^{-2x}}{2}$$

Hence Proved

ii) $\operatorname{sech}^2 x = 1 - \tanh^2 x$

$$\left(\frac{2}{e^{x}+e^{-x}}\right)^2 = 1 - \left(\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right)^2$$

$$\frac{4}{(e^{x}+e^{-x})^2} = \frac{(e^{x}+e^{-x})^2 - (e^{x}-e^{-x})^2}{(e^{x}+e^{-x})^2}$$

$$'' = \frac{\cancel{e^{2x}} + \cancel{e^{-2x}} + 2e^{x}e^{-x} - \cancel{e^{2x}} - \cancel{e^{-2x}} + 2e^{x}e^{-x}}{(e^{x}+e^{-x})^2}$$

$$'' = \frac{4e^{x}e^{-x}}{(e^{x}+e^{-x})^2} = \frac{4e^{x-x}}{(e^{x}+e^{-x})^2}$$

$$'' = \frac{4e^{0}}{(e^{x}+e^{-x})^2} = \frac{4}{(e^{x}+e^{-x})^2} \quad \because e^{0}=1$$

Hence Proved

iii) $\operatorname{csch}^2 x = \coth^2 x - 1$

$$\left(\frac{2}{e^{x}-e^{-x}}\right)^2 = \left(\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\right)^2 - 1$$

$$\frac{4}{(e^{x}-e^{-x})^2} = \frac{(e^{x}+e^{-x})^2-(e^{x}-e^{-x})^2}{(e^{x}-e^{-x})^2}$$

$$'' = \frac{\cancel{e^{2x}}+\cancel{e^{-2x}}+2e^{x}e^{-x}-\cancel{e^{2x}}-\cancel{e^{-2x}}+2e^{x}e^{-x}}{(e^{x}-e^{-x})^2}$$

$$'' = \frac{4e^{x}e^{-x}}{(e^{x}-e^{-x})^2}$$

$$'' = \frac{4e^{x-x}}{(e^{x}-e^{-x})^2}$$

$$'' = \frac{4e^{0}}{(e^{x}-e^{-x})^2}$$

$$'' = \frac{4}{(e^{x}-e^{-x})^2} \qquad \because e^0 = 1$$

Hence Proved

Q9:- Determine whether the given function f is even or odd.

Using

* $f(-x) = f(x)$

Then function is even

* $f(-x) = -f(x)$

Then function is odd

* $f(-x) \neq f(x) \neq -f(x)$

Then function is neither even nor odd

i) $f(x) = x^3 + x$

Put $x = -x$

$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x)$$

$$= -f(x)$$

The function is odd

ii) $f(x) = (x+2)^2$

Put $x = -x$

$f(-x) = (-x+2)^2$

$f(-x) = (-(x-2))^2 = (x-2)^2$

$f(-x) \neq (x+2)^2$

$f(-x) \neq f(x)$

The function is neither even nor odd.

iii) $f(x) = x\sqrt{x^2+5}$

Put $x = -x$

$f(-x) = -x\sqrt{(-x)^2+5}$

$f(-x) = -x\sqrt{x^2+5}$

$f(-x) = -f(x)$

The function is odd.

iv) $f(x) = \dfrac{x-1}{x+1}$

Put $x = -x$

$f(-x) = \dfrac{-x-1}{-x+1}$

$$f(-x) = \frac{\cancel{-}(x+1)}{\cancel{-}(x-1)}$$

$$f(-x) = \frac{x+1}{x-1}$$

$$f(-x) \neq f(x)$$

The function is neither even nor odd.

v) $f(x) = x^{2/3} + 6$

Put $x = -x$

$$f(-x) = (-x)^{2/3} + 6$$

$$f(-x) = ((-x)^2)^{\frac{1}{3}} + 6$$

$$f(-x) = x^{\frac{2}{3}} + 6$$

$$f(-x) = f(x)$$

The function is even.

vi)

$$f(x) = \frac{x^3 - x}{x^2 + 1}$$

Put $x = -x$

$$f(-x) = \frac{(-x)^3 - (-x)}{(-x)^2 + 1}$$

$$f(-x) = \frac{-x^3 + x}{x^2+1}$$

$$f(-x) = -\frac{(x^3 - x)}{x^2+1}$$

$$f(-x) = -f(x)$$

The function is odd.