Al-Huda Science Academy

Mathematics Notes

Class 12
Exercise 1.2

Prepared by:
Sir Muhammad Usman Ali

Exercise 1.2

Q1:- The real valued functions f and g are defined below.

a) $fog(x)$ b) $gof(x)$

c) $fof(x)$ d) $gog(x)$

i) $f(x) = 2x+1$, $g(x) = \dfrac{3}{x-1}$

a) $fog(x)$

$= f(g(x))$

$= f\left(\dfrac{3}{x-1}\right)$ Putting value of $g(x)$

$= 2\left(\dfrac{3}{x-1}\right)+1$ Putting value in $f(x)$.

$= \dfrac{6}{x-1} + 1$

$= \dfrac{6+x-1}{x-1} = \dfrac{x+5}{x-1}$

b) $g \circ f(x)$

$= g(f(x))$

$= g(2x+1)$ Putting value of $f(x)$

$= \dfrac{3}{2x+\cancel{1}-\cancel{1}}$ Putting value in $g(x)$.

$= \dfrac{3}{2x}$

c) $f \circ f(x)$

$= f(f(x))$

$= f(2x+1)$ Putting value of $f(x)$.

$= 2(2x+1)+1$ Putting value in $f(x)$.

$= 4x+2+1$

$= 4x+3$

d) $g \circ g(x)$

$= g(g(x))$

$= g\left(\dfrac{3}{x-1}\right)$ Putting value of $g(x)$

$= \dfrac{3}{\frac{3}{x-1}-1}$ Putting value in $g(x)$

$$= \frac{3}{\frac{3-(x-1)}{x-1}}$$

$$= \frac{3(x-1)}{3-x+1}$$

$$= \frac{3(x-1)}{4-x}$$

ii) $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2}$

a) $f o g(x)$

$= f(g(x))$

$= f\left(\frac{1}{x^2}\right)$ Putting the value of $g(x)$

$= \sqrt{\frac{1}{x^2} + 1}$ Putting the value in $f(x)$

$$= \sqrt{\frac{1+x^2}{x^2}}$$

$$= \frac{\sqrt{1+x^2}}{x^1} \Rightarrow \frac{\sqrt{1+x^2}}{x}$$

b) $gof(x)$

$= g(f(x))$

$= g(\sqrt{x+1})$ Putting the value of $f(x)$

$= \frac{1}{(\sqrt{x+1})^2}$ Putting the value in $g(x)$

$= \frac{1}{x+1}$

c) $fof(x)$

$= f(f(x))$

$= f(\sqrt{x+1})$ Putting the value of $f(x)$

$= \sqrt{\sqrt{x+1}+1}$ Putting the value in $f(x)$

d) $gog(x)$

$= g(g(x)$

$= g\left(\frac{1}{x^2}\right)$ Putting the value of $g(x)$

$= \frac{1}{\left(\frac{1}{x^2}\right)^2}$ Putting the value in $g(x)$

$= \frac{1}{\frac{1}{x^4}}$

$= x^4$

iii) $f(x) = \dfrac{1}{\sqrt{x-1}}$, $g(x) = (x^2+1)^2$

a) $f o g(x)$

$= f(g(x))$

$= f(x^2+1)^2$ Putting the value of $g(x)$.

$= \dfrac{1}{\sqrt{(x^2+1)^2 - 1}}$ Putting the value in $f(x)$.

$= \dfrac{1}{\sqrt{x^4+2x^2+\cancel{1}-\cancel{1}}}$

$= \dfrac{1}{\sqrt{x^4+2x^2}} = \dfrac{1}{\sqrt{x^2(x^2+2)}}$

$= \dfrac{1}{x\sqrt{x^2+2}}$

b) $g o f(x)$

$= g(f(x))$

$= g\left(\dfrac{1}{\sqrt{x-1}}\right)$ Putting the value of $f(x)$

$= \left(\left(\dfrac{1}{\sqrt{x-1}}\right)^x + 1\right)^2$ Putting the value in $g(x)$.

$$= \left(\frac{1}{x-1}+1\right)^2$$

$$= \left(\frac{\cancel{1}+x-\cancel{1}}{x-1}\right)^2$$

$$= \left(\frac{x}{x-1}\right)^2$$

c) $f \circ f(x)$

$$= f(f(x))$$

$$= f\left(\frac{1}{\sqrt{x-1}}\right)$$ Putting the value of $f(x)$

$$= \frac{1}{\sqrt{\frac{1}{\sqrt{x-1}}-1}}$$ Putting the value in $f(x)$.

$$= \frac{1}{\sqrt{\frac{1-\sqrt{x-1}}{\sqrt{x-1}}}}$$

$$= \sqrt{\frac{\sqrt{x-1}}{1-\sqrt{x-1}}}$$

d) $gog(x)$

$= g(g(x))$

$= g((x^2+1)^2)$ Putting the value of $g(x)$

$= [\{(x^2+1)^2\}^2+1]^2$ Putting the value in $g(x)$

$= [(x^2+1)^4+1]^2$

iv) $f(x) = 3x^4 - 2x^2$, $g(x) = \frac{2}{\sqrt{x}}$

a) $fog(x)$

$= f(g(x))$

$= f\left(\frac{2}{\sqrt{x}}\right)$ Putting the value of $g(x)$

$= 3\left(\frac{2}{\sqrt{x}}\right)^4 - 2\left(\frac{2}{\sqrt{x}}\right)^2$ Putting the value in $f(x)$

$= 3\left(\frac{16}{x^2}\right) - 2\left(\frac{4}{x}\right)$

$$= \frac{48}{x^2} - \frac{8}{x}$$

$$= \frac{48-8x}{x^2} = \frac{8(6-x)}{x^2}$$

b) $g \circ f(x)$

$= g(f(x))$

$= g(3x^4 - 2x^2)$ Putting the value of $f(x)$

$= \frac{2}{\sqrt{3x^4 - 2x^2}}$ Putting the value in $g(x)$

$= \frac{2}{x\sqrt{3x^2 - 2}}$

c) $f \circ f(x)$

$= f(f(x))$

$= f(3x^4 - 2x^2)$ Putting the value of $f(x)$

$= 3(3x^4 - 2x^2)^4 - 2(3x^4 - 2x^2)^2$ Putting the value in $f(x)$

$= (3x^4 - 2x^2)^2 [3(3x^4 - 2x^2)^2 - 2]$

d) $gog(x)$

$= g(g(x))$

$= g\left(\frac{2}{\sqrt{x}}\right)$ Putting the value of $g(x)$

$= \frac{2}{\sqrt{\frac{2}{\sqrt{x}}}}$ Putting the value in $g(x)$.

$= \frac{2\sqrt{\sqrt{x}}}{\sqrt{2}}$

$= \sqrt{2}\sqrt{\sqrt{x}}$ $\because \frac{2}{\sqrt{2}} = \sqrt{2}$

$= \sqrt{2\sqrt{x}}$

Q2:- For Real value functions, f defined below, find

a) $f^{-1}(x)$ b) $f^{-1}(-1)$ and verify $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

i) $f(x) = -2x+8$

a) As,

$$y = f(x) \Rightarrow f^{-1}(y) = x$$

$$y = -2x+8$$

$$2x = 8-y$$

$$x = \frac{8-y}{2}$$

$$f^{-1}(y) = \frac{8-y}{2} \quad \because x = f^{-1}(y)$$

Replace 'y' with 'x'

$$f^{-1}(x) = \frac{8-x}{2}$$

b) $f^{-1}(-1)$

$$f^{-1}(-1) = \frac{8-(-1)}{2} = \frac{8+1}{2} = \frac{9}{2}$$

Verification

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

When $f(x) = -2x+8$, $f^{-1}(x) = \frac{8-x}{2}$

$f(f^{-1}(x)) = f\left(\frac{8-x}{2}\right)$ Putting the value of $f^{-1}(x)$

$= -\cancel{2}\left(\frac{8-x}{\cancel{2}}\right)+8$ Putting the value in $f(x)$

$= -\cancel{8}+x+\cancel{8}$

$= x$

Now,

$f^{-1}(f(x)) = f^{-1}(-2x+8)$ Putting the value of $f(x)$

$= \frac{8-(-2x+8)}{2}$ Putting the value in $f^{-1}(x)$.

$= \frac{\cancel{8}+2x-\cancel{8}}{2}$

$= \frac{\cancel{2}x}{\cancel{2}} = x$

Hence, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

ii) $f(x) = 3x^3 + 7$

a) $f^{-1}(x)$

As, $y = f(x) \Rightarrow f^{-1}(y) = x$

$$y = 3x^3 + 7$$

$$y - 7 = 3x^3$$

$$\frac{y-7}{3} = x^3$$

$$\left(\frac{y-7}{3}\right)^{\frac{1}{3}} = x$$

$$\Rightarrow \quad x = \left(\frac{y-7}{3}\right)^{\frac{1}{3}}$$

$$f^{-1}(y) = \left(\frac{y-7}{3}\right)^{\frac{1}{3}} \quad \because x = f^{-1}(y)$$

Replace 'y' with 'x'

$$f^{-1}(x) = \left(\frac{x-7}{3}\right)^{\frac{1}{3}}$$

b) $f^{-1}(-1)$

$$f^{-1}(-1) = \left(\frac{-1-7}{3}\right)^{\frac{1}{3}} = \left(\frac{-8}{3}\right)^{\frac{1}{3}}$$

$$= \frac{-2}{3^{1/3}}$$

Verification

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

when $f(x) = 3x^3+7$, $f^{-1}(x) = \left(\frac{x-7}{3}\right)^{\frac{1}{3}}$

$$f(f^{-1}(x)) = f\left(\frac{x-7}{3}\right)^{\frac{1}{3}}$$ Putting the value of $f^{-1}(x)$

$$= 3\left[\left(\frac{x-7}{3}\right)^{\frac{1}{\cancel{3}}}\right]^{\cancel{3}}+7$$ Putting the value in $f(x)$

$$= \cancel{3}\left(\frac{x-7}{\cancel{3}}\right)+7$$

$$= x-\cancel{7}+\cancel{7}$$

$$= x$$

$$f^{-1}(f(x)) = f^{-1}(3x^3+7)$$ Putting the value of $f(x)$

$$= \left(\frac{3x^3+\cancel{7}-\cancel{7}}{3}\right)^{\frac{1}{3}}$$ Putting the value in $f^{-1}(x)$

$$= \left(\frac{\cancel{3}x^3}{\cancel{3}}\right)^{\frac{1}{3}}$$

$$= (x^{\cancel{3}})^{\frac{1}{\cancel{3}}} = x$$

Hence,

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

iii) $f(x) = (-x+9)^3$

a) $f^{-1}(x)$

As, $y = f(x) \Rightarrow f^{-1}(y) = x$

$$y = (-x+9)^3$$

$$y^{\frac{1}{3}} = -x+9$$

$$x = 9 - y^{\frac{1}{3}}$$

$$f^{-1}(y) = 9 - y^{\frac{1}{3}} \quad \because x = f^{-1}(y)$$

Replace 'y' with 'x'

$$f^{-1}(x) = 9 - x^{\frac{1}{3}}$$

b) $f^{-1}(-1)$

$$f^{-1}(-1) = 9 - (-1)^{\frac{1}{3}}$$

$$= 9 - (-1)$$

$$= 9 + 1 = 10$$

<u>Verification</u>

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

when $f(x) = (-x+9)^3$, $f^{-1}(x) = 9 - x^{\frac{1}{3}}$

$f(f^{-1}(x)) = f(9 - x^{\frac{1}{3}})$ Putting the value of $f^{-1}(x)$

$= (-(9 - x^{\frac{1}{3}}) + 9)^3$ Putting the value in $f(x)$

$$= (\cancel{-9} + x^{\frac{1}{3}} + \cancel{9})^3$$

$$= (x^{\frac{1}{\cancel{3}}})^{\cancel{3}}$$

$$= x$$

$f^{-1}(f(x)) = f^{-1}((-x+9)^3)$ Putting the value of $f(x)$

$= 9-((-x+9)^3)^{\frac{1}{3}}$ Putting the value in $f^{-1}(x)$

$= 9-(-x+9)$

$= \cancel{9}+x-\cancel{9}$

$= x$

Hence,

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

iv) $f(x) = \dfrac{2x+1}{x-1}$

a) $f^{-1}(x)$

As, $y = f(x) \Rightarrow f^{-1}(y) = x$

$y = \dfrac{2x+1}{x-1}$

$y(x-1) = 2x+1$

$xy - y = 2x+1$

$xy - 2x = y+1$

$x(y-2) = y+1$

$x = \dfrac{y+1}{y-2}$

$$f^{-1}(y) = \frac{y+1}{y-2} \qquad \because x = f^{-1}(y)$$

Replace 'y' with 'x'

$$f^{-1}(x) = \frac{x+1}{x-2}$$

b) $f^{-1}(-1)$

$$f^{-1}(-1) = \frac{-1+1}{-1-2} = \frac{0}{-3} = 0$$

Verification

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

when $f(x) = \frac{2x+1}{x-1}$, $f^{-1}(x) = \frac{x+1}{x-2}$

$$f(f^{-1}(x)) = f\left(\frac{x+1}{x-2}\right) \qquad \text{Putting the value of } f^{-1}(x)$$

$$= \frac{2\left(\frac{x+1}{x-2}\right)+1}{\left(\frac{x+1}{x-2}\right)-1} \qquad \text{Putting the value in } f(x)$$

$$= \frac{\frac{2x+\cancel{2}+x-\cancel{2}}{\cancel{x-2}}}{\frac{\cancel{x}+1-\cancel{x}+2}{\cancel{x-2}}} \cancel{= 3x+3}$$

$$= \frac{\cancel{3}x}{\cancel{3}} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+1}{x-1}\right)$$ Putting the value of $f(x)$

$$= \frac{\frac{2x+1}{x-1}+1}{\frac{2x+1}{x-1}-2}$$ Putting the value in $f^{-1}(x)$

$$= \frac{\frac{2x+\cancel{1}+x-\cancel{1}}{\cancel{x-1}}}{\frac{\cancel{2x}+1-\cancel{2x}+2}{\cancel{x-1}}}$$

$$= \frac{\cancel{3}x}{\cancel{3}}$$

$$= x$$

Hence,

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

Q3:- Without finding inverse, state the domain and range of f^{-1}.

i) $f(x) = \sqrt{x+2}$

If $x \geq -2$

Domain $f(x) = [-2, \infty)$

Range $f(x) = [0, \infty)$

Hence,

Domain $f^{-1}(x) = [0, \infty)$

Range $f^{-1}(x) = [-2, \infty)$

ii) $f(x) = \dfrac{x-1}{x-4}, x \neq 4$

If $x \neq 4$

Domain $f(x) = R - \{4\}$

Range $f(x) = R - \{1\}$

Hence,

Domain $f^{-1}(x) = R - \{1\}$

Range $f^{-1}(x) = R - \{4\}$

iii) $f(x) = \dfrac{1}{x+3}, \quad x \neq -3$

If $x \neq -3$

Domain $f(x) = R - \{-3\}$

Range $f(x) = R - \{0\}$

Hence,

Domain $f^{-1}(x) = R - \{0\}$

Range $f^{-1}(x) = R - \{-3\}$

iv) $f(x) = (x-5)^2, \quad x \geq 5$

If $x \geq 5$

Domain $f(x) = [5, \infty)$

Range $f(x) = [0, \infty)$

Hence,

Domain $f^{-1}(x) = [0, \infty)$

Range $f^{-1}(x) = [5, \infty)$