Al-Huda Science Academy

Mathematics Notes

Class 12
Exercise 1.3

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Exercise 1.3

Q1:- Evaluate each limit using theorems of limits.

i) $\lim_{x\to 3}(2x+4)$

$= \lim_{x\to 3}(2x) + \lim_{x\to 3}(4)$ ∵ Using Sum Theorem

$= 2\lim_{x\to 3}(x) + 4$ ∵ $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$ and $\lim_{x\to a} c = c$

$= 2(3)+4$

$= 6+4$

$= 10$

ii) $\lim_{x\to 1}(3x^2-2x+4)$

Using Sum & Difference theorem

$= \lim_{x\to 1}(3x^2) - \lim_{x\to 1}(2x) + \lim_{x\to}(4)$

$= 3\lim_{x\to 1}(x^2) - 2\lim_{x\to 1}(x) + 4$ $\quad \because \lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$

and $\lim_{x\to a} c = c$

$= 3(1)^2 - 2(1) + 4$

$= 3 - 2 + 4$

$= 5$

iii) $\lim_{x\to 3}\sqrt{x^2+x+4}$

$= \lim_{x\to 3}(x^2+x+4)^{\frac{1}{2}}$

$= \left[\lim_{x\to 3}(x^2+x+4)\right]^{\frac{1}{2}}$ $\quad \because \lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$

$= \left[\lim_{x\to 3}(x^2) + \lim_{x\to 3}(x) + \lim_{x\to 3}(4)\right]^{\frac{1}{2}}$ $\quad \because$ Sum Theorem

$= ((3)^2 + 3 + 4)^{\frac{1}{2}}$ $\quad \because \lim_{x\to a} c = c$

$= (9 + 7)^{\frac{1}{2}}$

$= (16)^{\frac{1}{2}}$

$= 4$

iv) $\lim_{x \to 2} x\sqrt{x^2-4}$

$= \lim_{x \to 2} x . \lim_{x \to 2} \sqrt{x^2-4}$ $\because$ Product Rule

$= (2)\left[\lim_{x \to 2}(x^2-4)\right]^{\frac{1}{2}}$ $\because \lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$

$= (2)\left[\lim_{x \to 2}(x^2) - \lim_{x \to 2}(4)\right]^{\frac{1}{2}}$ $\because$ Difference Rule

$= (2)\left[(2)^2 - 4\right]^{\frac{1}{2}}$ $\because \lim_{x \to a} c = c$

$= (2)\left[4-4\right]^{\frac{1}{2}}$

$= 2(0)^{\frac{1}{2}}$

$= 2(0) = 2$

v) $\lim_{x \to 2}\left(\sqrt{x^3+1} - \sqrt{x^2+5}\right)$

$= \lim_{x \to 2}\sqrt{x^3+1} - \lim_{x \to 2}\sqrt{x^2+5}$ Difference Rule

$= \left[\lim_{x \to 2}(x^3+1)\right]^{\frac{1}{2}} - \left[\lim_{x \to 2}(x^2+5)\right]^{\frac{1}{2}}$ $\because \lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$

$= \left[\lim_{x \to 2}(x^3) + \lim_{x \to 2}(1)\right]^{\frac{1}{2}} - \left[\lim_{x \to 2}(x^2) + \lim_{x \to 2}(5)\right]^{\frac{1}{2}}$

Sum Rule

$= \left[(2)^3+1\right]^{\frac{1}{2}} - \left[(2)^2+5\right]^{\frac{1}{2}}$

$= (8+1)^{\frac{1}{2}} - (4+5)^{\frac{1}{2}}$

$= (9)^{\frac{1}{2}} - (9)^{\frac{1}{2}}$

$= 3-3$

$= 0$

vi) $\lim\limits_{x\to -2} \dfrac{2x^3+5x}{3x-2}$

$= \dfrac{\lim\limits_{x\to -2}(2x^3+5x)}{\lim\limits_{x\to -2}(3x-2)}$ $\because$ Quotient Rule

$= \dfrac{\lim\limits_{x\to -2}(2x^3)+\lim\limits_{x\to -2}(5x)}{\lim\limits_{x\to -2}(3x)-\lim\limits_{x\to -2}(2)}$ $\because$ Sum and Difference Rule

$= \dfrac{2\lim\limits_{x\to -2}(x^3)+5\lim\limits_{x\to -2}(x)}{3\lim\limits_{x\to -2}(x)-2}$ $\because \lim\limits_{x\to a} cf(x) = c\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} c = c$

$= \dfrac{2(-2)^3+5(-2)}{3(-2)-2} = \dfrac{2(-8)-10}{-6-2}$

$= \dfrac{-26}{-8} = \dfrac{13}{4}$

Q2:- Evaluate each limit using algebraic techniques.

Not exists in Limit $\because \frac{0}{0}, \frac{\infty}{\infty}$

i) $\lim\limits_{x \to 1} \frac{x^3 - x}{x+1}$

$= \lim\limits_{x \to -1} \frac{x(x^2-1)}{x+1}$

$= \lim\limits_{x \to -1} \frac{x\cancel{(x+1)}(x-1)}{\cancel{x+1}}$

$= \lim\limits_{x \to -1} x(x-1) = \lim\limits_{x \to -1} (x) . \lim\limits_{x \to -1} (x-1)$

$= -1(-1-1)$

$= -1(-2)$

$= 2$

ii) $\lim\limits_{x \to 1} \frac{3x^3 + 4x}{x^2 + x}$

$= \lim\limits_{x \to 1} \frac{\cancel{x}(3x^2+4)}{\cancel{x}(x+1)}$

$= \lim\limits_{x \to 1} \frac{3x^2+4}{x+1}$

$= \frac{\lim\limits_{x \to 1} (3x^2+4)}{\lim\limits_{x \to 1} (x+1)}$ $\because$ Quotient Rule

$$= \frac{3(1)^2+4}{1+1} = \frac{3+4}{2} = \frac{7}{2}$$

iii) $\lim_{x \to 2} \frac{x^3-8}{x^2+x-6}$

$$= \lim_{x \to 2} \frac{x^3-2^3}{x^2+3x-2x-6}$$

$$= \lim_{x \to 2} \frac{(x-2)[(x)^2+(x)(2)+(2)^2]}{x(x+3)-2(x+3)}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+3)}$$

$$= \lim_{x \to 2} \frac{x^2+2x+4}{x+3}$$

$$= \frac{\lim_{x \to 2}(x^2+2x+4)}{\lim_{x \to 2}(x+3)}$$ $\because$ Quotient Rule

$$= \frac{(2)^2+2(2)+4}{2+3}$$

$$= \frac{4+4+4}{5} = \frac{12}{5}$$

iv) $\lim_{x \to 1} \frac{x^3-3x^2+3x-1}{x^3-x}$

$= \lim_{x \to 1} \frac{x^3-1-3x^2+3x}{x(x^2-1)}$

$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)-3x(x-1)}{x(x+1)(x-1)}$

$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1-3x)}{x(x+1)\cancel{(x-1)}}$

$= \lim_{x \to 1} \frac{x^2-2x+1}{x(x+1)}$

$= \frac{\lim_{x \to 1}(x^2-2x+1)}{\lim_{x \to 1} x(x+1)}$ $\because$ Quotient Rule

$= \frac{(1)^2-2(1)+1}{(1)(1+1)} = \frac{1-2+1}{2}$

$= \frac{0}{2} = 0$

v) $\lim_{x \to -1} \frac{x^3+x^2}{x^2-1}$

$= \lim_{x \to -1} \frac{x^2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$= \lim_{x \to -1} \frac{x^2}{x-1}$

$= \frac{(-1)^2}{-1-1} = \frac{1}{-2}$

vi) $\lim_{x \to 4} \frac{2x^2-32}{x^3-4x^2}$

$= \lim_{x \to 4} \frac{2(x^2-16)}{x^2(x-4)}$

$= \lim_{x \to 4} \frac{2(x^2-4^2)}{x^2(x-4)}$

$= \lim_{x \to 4} \frac{2(x+4)\cancel{(x-4)}}{x^2\cancel{(x-4)}}$

$= \frac{2\lim_{x \to 4}(x+4)}{\lim_{x \to 4}(x^2)}$

$= \frac{2(4+4)}{(4)^2} = \frac{2(8)}{16} = \frac{16}{16} = 1$

vii) $\lim_{x \to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

$= \lim_{x \to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \times \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}$

$= \lim_{x \to 2} \frac{(\sqrt{x})^2-(\sqrt{2})^2}{(x-2)(\sqrt{x}+\sqrt{2})}$

$= \lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x}+\sqrt{2})}$

$= \lim_{x \to 2} \frac{1}{\sqrt{x}+\sqrt{2}}$

$= \frac{1}{\sqrt{2}+\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}$

viii) $\lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \times \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

ix) $\lim_{x \to a} \frac{x^n - a^n}{x^m - a^m}$

Using Binomial theorem.

$$= \lim_{x \to a} \frac{\cancel{(x+a)}(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \cdots + a^{n-1})}{\cancel{(x+a)}(x^{m-1} + x^{m-2}a + x^{m-3}a^2 + \cdots + a^{m-1})}$$

$$= \frac{\lim_{x\to a}\left(x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots+a^{n-1}\right)}{\lim_{x\to a}\left(x^{m-1}+x^{m-2}a+x^{m-3}a^2+\cdots+a^{m-1}\right)}$$

$$= \frac{a^{n-1}+a^{n-2}a+a^{n-3}a^2+\cdots+a^{n-1}}{a^{m-1}+a^{m-2}a+a^{n-3}a^2+\cdots+a^{m-1}}$$

$$= \frac{a^{n-1}+a^{n-1}+a^{n-1}+\cdots+a^{n-1}}{a^{m-1}+a^{m-1}+a^{m-1}+\cdots+a^{m-1}}$$

$$= \frac{na^{n-1}}{ma^{m-1}}$$

$$= \frac{n}{m}a^{n-\cancel{1}-m+\cancel{1}}$$

$$= \frac{n}{m}a^{n-m}$$

Q3:- Evaluate the following limits

i) $\lim_{x \to 0} \frac{\sin 7x}{x}$

$= \lim_{x \to 0} \frac{\sin 7x}{7x} \times 7$

$= 1 \times 7$ $\quad \because \lim_{x \to 0} \frac{\sin x}{x} = 1$

$= 7$

ii) $\lim_{x \to 0} \frac{\sin x^\circ}{x}$

Let

$$1^\circ = \frac{\pi}{180} \text{ radians}$$

Multiply by 'x'

$$x^\circ = \frac{\pi x}{180}$$

Then

$$= \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{x}$$

$$= \lim_{x \to 0} \frac{\sin \frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$$

$$= 1 \times \frac{\pi}{180} \quad \because \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$= \frac{\pi}{180}$$

iii) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\sin\theta}$

$$= \lim_{\theta \to 0} \frac{1-\cos\theta}{\sin\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \lim_{\theta \to 0} \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1+\cos\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{\sin 0^\circ}{1+\cos 0^\circ}$$

$$= \frac{0}{1+1}$$

$$= 0$$

iv) $\lim_{x \to \pi} \frac{\sin x}{\pi - x}$

Let

$\pi - x = \theta$ — (1)

and $x = \pi - \theta$

When $x \to \pi$

Put $x = \pi$ in (1)

$\pi - \pi = \theta$

$0 = \theta$

So, $\theta \to 0$

Then

$$= \lim_{\theta \to 0} \frac{\sin(\pi - \theta)}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \quad \because \sin(\pi-\theta) = \sin\theta$$

$= 1$ $\quad \because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

v) $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$

$= \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{bx}{\sin bx} \times \frac{a}{b}$

$= 1 \times 1 \times \frac{a}{b}$ $\quad \because \lim_{x \to 0} \frac{\sin x}{x} = 1$

$= \frac{a}{b}$

vi) $\lim_{x \to 0} \frac{x}{\tan x}$

$= \lim_{x \to 0} \frac{x}{\frac{\sin x}{\cos x}}$ $\quad \because \tan x = \frac{\sin x}{\cos x}$

$= \lim_{x \to 0} \frac{x}{\sin x} \times \cos x$

$= \frac{\lim_{x \to 0} \cos x}{\lim_{x \to 0} \frac{\sin x}{x}}$

$= \frac{\cos 0}{1}$ $\quad \because \lim_{x \to 0} \frac{\sin x}{x} = 1$

$= 1$

vii) $\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$

$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$

$= 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$

$= 2(1)^2$ $\quad \because \lim_{x\to 0}\frac{\sin x}{x} = 1$

$= 2$

viii) $\lim_{x\to 0} \frac{1-\cos x}{\sin^2 x}$

$= \lim_{x\to 0} \frac{1-\cos x}{\sin^2 x} \times \frac{1+\cos x}{1+\cos x}$

$= \lim_{x\to 0} \frac{1-\cos^2 x}{\sin^2 x(1+\cos x)}$

$= \lim_{x\to 0} \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x}(1+\cos x)}$

$= \lim_{x\to 0} \frac{1}{1+\cos x}$

$= \frac{1}{1+\cos 0^\circ} = \frac{1}{1+1} = \frac{1}{2}$

ix) $\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{\sin^2\theta}{\theta^2} \times \theta$

$= \lim_{\theta \to 0} \left(\frac{\sin\theta}{\theta}\right)^2 \times \theta$

$= (1)^2 \times 0$

$= 0$

x) $\lim_{x \to 0} \frac{\sec x - \cos x}{x}$

$= \lim_{x \to 0} \frac{\frac{1}{\cos x} - \cos x}{x}$

$= \lim_{x \to 0} \frac{\frac{1-\cos^2 x}{\cos x}}{x}$

$= \lim_{x \to 0} \frac{\sin^2 x}{x \cos x}$

$= \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \frac{x}{\cos x}$

$= \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 \times \lim_{x \to 0} \frac{x}{\cos x}$

$$= (1)^2 \times \frac{0}{\cos 0} \qquad \because \lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \frac{0}{1}$$

$$= 0$$

xi) $\lim_{\theta\to 0} \frac{1-\cos p\theta}{1+\cos q\theta}$

$$= \lim_{\theta\to 0} \frac{\cancel{2}\sin^2 \frac{p\theta}{2}}{\cancel{2}\sin^2 \frac{q\theta}{2}}$$

$$= \lim_{\theta\to 0} \frac{\sin^2 \frac{p\theta}{2}}{\frac{p^2\theta^2}{4}} \times \frac{\frac{q^2\theta^2}{4}}{\sin^2 \frac{q\theta}{2}} \times \frac{\frac{p^2\cancel{\theta^2}}{\cancel{4}}}{\frac{q^2\cancel{\theta^2}}{\cancel{4}}}$$

$$= \lim_{\theta\to 0} \left(\frac{\sin \frac{p\theta}{2}}{\frac{p\theta}{2}}\right)^2 \times \left(\frac{\frac{q\theta}{2}}{\sin \frac{q\theta}{2}}\right)^2 \times \frac{p^2}{q^2}$$

$$= (1)^2 \times (1)^2 \times \frac{p^2}{q^2} \qquad \because \lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \frac{p^2}{q^2}$$

xii) $\lim\limits_{\theta \to 0} \dfrac{\tan\theta - \sin\theta}{\sin^3\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{\frac{\sin\theta}{\cos\theta} - \sin\theta}{\sin^3\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{\frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta}}{\sin^3\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{\cancel{\sin\theta}(1-\cos\theta)}{\cos\theta\,\cancel{\sin^3}^{2}\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{1-\cos\theta}{\cos\theta\sin^2\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{1-\cos\theta}{\cos\theta\sin^2\theta} \times \dfrac{1+\cos\theta}{1+\cos\theta}$

$= \lim\limits_{\theta \to 0} \dfrac{1-\cos^2\theta}{\cos\theta\sin^2\theta(1+\cos\theta)}$

$= \lim\limits_{\theta \to 0} \dfrac{\cancel{\sin^2\theta}}{\cos\theta\,\cancel{\sin^2\theta}(1+\cos\theta)}$

$= \lim\limits_{\theta \to 0} \dfrac{1}{\cos\theta(1+\cos\theta)}$

$= \dfrac{1}{\cos 0^\circ(1+\cos 0^\circ)} = \dfrac{1}{1(1+1)} = \dfrac{1}{2}$

Q4:- Express each limit in terms of e.

i) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

$= \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{2}$

$= \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{2}$

$= e^{2} \qquad \because \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

ii) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

$= \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{2}}$

$= \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]^{\frac{1}{2}}$

$= e^{\frac{1}{2}} \qquad \because \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

iii) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n$

$= \lim_{n\to\infty}\left[\left(1+\left(\frac{-1}{n}\right)\right)^{-n}\right]^{-1}$

$= \left[\lim_{n\to\infty}\left(1+\left(-\frac{1}{n}\right)\right)^{-n}\right]^{-1}$

$= e^{-1}$ $\quad \because \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

$= \frac{1}{e}$

iv) $\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^n$

$= \lim_{n\to\infty}\left[\left(1+\frac{1}{3n}\right)^{3n}\right]^{1/3}$

$= \left[\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{3n}\right]^{1/3}$

$= e^{1/3}$ $\quad \because \lim_{n\to\infty}\left(1+\frac{1}{cn}\right)^{cn} = e$

v) $\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{n}$

$= \lim_{n\to\infty}\left[\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^{4}$

$= \left[\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right]^{4}$

$= e^{4}$ $\quad \because \lim_{n\to\infty}\left(1+\frac{c}{n}\right)^{\frac{n}{c}}$

vi) $\lim_{x\to 0}(1+3x)^{\frac{2}{x}}$

$= \lim_{x\to 0}(1+3x)^{\frac{2\times 3}{3x}}$

$= \left[\lim_{x\to 0}(1+3x)^{\frac{1}{3x}}\right]^{6}$

$= e^{6}$ $\quad \because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$

vii) $\lim_{x\to 0}(1+2x^{2})^{\frac{1}{x^{2}}}$

$= \lim_{x\to 0}(1+2x^{2})^{\frac{1}{2x^{2}}\times 2}$

$$=\left[\lim_{x\to 0}\left(1+2x^2\right)^{\frac{1}{2x^2}}\right]^2$$

$$= e^2 \qquad \because \lim_{x\to 0}\left(1+cx^2\right)^{\frac{1}{cx^2}} = e$$

viii) $$\lim_{h\to 0}\left(1-2h\right)^{\frac{1}{h}}$$

$$=\lim_{h\to 0}\left(\left(1+(-2h)\right)^{\frac{1}{-2h}}\right)^{-2}$$

$$=\left[\lim_{h\to 0}\left(1+(-2h)\right)^{\frac{1}{-2h}}\right]^{-2}$$

$$= e^{-2}$$

$$= \frac{1}{e^2}$$

ix) $$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$$

$$=\lim_{x\to\infty}\left[\left(\frac{1+x}{x}\right)^{-1}\right]^x$$

$$=\lim_{x\to\infty}\left[\left(\frac{1}{x}+\frac{\cancel{x}}{\cancel{x}}\right)^x\right]^{-1}$$

$$=\left[\lim_{x\to\infty}\left(\frac{1}{x}+1\right)^x\right]^{-1}$$

$$= \left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right]^{-1}$$

$$= e^{-1} \qquad \because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = e$$

$$= \frac{1}{e}$$

x) $\lim_{x\to 0} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x<0$

Let

$x = -t$

Then

$t \to 0$

Equation becomes,

$$= \lim_{t\to 0} \frac{e^{-\frac{1}{t}}-1}{e^{-\frac{1}{t}}+1}$$

$$= \lim_{t\to 0} \frac{\frac{1}{e^{1/t}}-1}{\frac{1}{e^{1/t}}+1}$$

$$= \frac{\frac{1}{e^{1/0}}-1}{\frac{1}{e^{1/0}}+1}$$

$$= \frac{\frac{1}{e^{\infty}}-1}{\frac{1}{e^{\infty}}+1}$$

$$= \frac{\frac{1}{\infty} - 1}{\frac{1}{\infty} + 1}$$

$$= \frac{0-1}{0+1}$$

$$= \frac{-1}{1} = -1$$

xi) $\lim\limits_{x \to 0} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, x > 0$

$$= \lim_{x \to 0} \frac{\frac{\cancel{e^{1/x}}}{\cancel{e^{1/x}}} - \frac{1}{e^{1/x}}}{\frac{\cancel{e^{1/x}}}{\cancel{e^{1/x}}} + \frac{1}{e^{1/x}}}$$

$$= \lim_{x \to 0} \frac{1 - \frac{1}{e^{1/x}}}{1 + \frac{1}{e^{1/x}}}$$

$$= \frac{1 - \frac{1}{e^{1/0}}}{1 + \frac{1}{e^{1/0}}}$$

$$= \frac{1 - \frac{1}{e^{\infty}}}{1 + \frac{1}{e^{\infty}}}$$

$$= \frac{1 - \frac{1}{\infty}}{1 + \frac{1}{\infty}}$$

$$= \frac{1-0}{1+0}$$

$$= \frac{1}{1}$$

$$= 1$$