Al-Huda Science Academy

Mathematics Notes

Class 12
Exercise 1.4

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Exercise 1.4

Q1:- Determine the left hand and right hand limit and then find the limits of the following functions at $x \to c$.

i) $f(x) = 2x^2 + x - 5$, $c = 1$

Answer

L.H.L

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x^2 + x - 5)$$

$$= 2(1)^2 + (1) - 5$$
$$= 2 + 1 - 5$$
$$= -2$$

R.H.L

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x^2 + x - 5)$$

$$= 2(1)^2 + (1) - 5 = 2 + 1 - 5$$
$$= -2$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} (2x^2 + x - 5)$$

$$= 2(1)^2 + (1) - 5$$

$$= 2 + 1 - 5$$

$$= -2$$

ii) $f(x) = \dfrac{x^2 - 9}{x - 3}, \quad c = -3$

Answer

L.H.L

$$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \frac{x^2 - 9}{x - 3}$$

$$= \lim_{x\to -3^-} \frac{x^2 - 3^2}{x - 3}$$

$$= \lim_{x\to -3^-} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to -3^-} (x+3)$$

$$= -3 + 3$$

$$= 0$$

R.H.L

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{x^2-9}{x-3}$$

$$= \lim_{x\to -3^+} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to -3^+} (x+3)$$

$$= -3+3 = 0$$

$$\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{x^2-9}{x-3}$$

$$= \lim_{x\to -3} \frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= \lim_{x\to -3} (x+3)$$

$$= -3+3$$
$$= 0$$

iii) $f(x) = |x-5|, c=5$

Answer

L.H.L

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} |x-5|$$

$$= |5-5|$$

$$= 0$$

R.H.L

$$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} |x-5|$$

$$= |5-5|$$

$$= 0$$

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} |x-5|$$

$$= |5-5|$$

$$= 0$$

Q2:- Discuss the continuity of $f(x)$ at $x=c$.

i) $f(x) = \begin{cases} 2x+5 & \text{if } x \leq 2, c=2 \\ 4x+1 & \text{if } x>2 \end{cases}$

Answer

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} (2x+5)$$

$$= 2(2)+5$$

$$= 4+5 = 9$$

L.H.L

x is less than 2

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x+5)$$

$$= 2(2)+5 = 4+5 = 9$$

R.H.L

x is greater than 2

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (4x+1)$$

$$= 4(2)+1 = 8+1 = 9$$

As, $\lim_{x\to 2} f(x) = L.H.L = R.H.L$

Therefore, $f(x)$ is continuous at $x=2$.

ii) $f(x) = \begin{cases} 3x-1 & \text{if } x<1 \\ 4 & \text{if } x=1 \\ 2x & \text{if } x>1 \end{cases}$, $c=1$

Answer

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} (4)$$

$$= 4$$

L.H.L

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (3x-1)$$

$$= 3(1)-1 = 3-1 = 2$$

R.H.L

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x)$$

$$= 2(1) = 2$$

As,

$$\lim_{x\to 1} f(x) \neq L.H.L = R.H.L$$

Therefore, $f(x)$ is discontinuous at $x=1$

iii) $f(x) = \begin{cases} 3x-1 & \text{if } x < 1, \ c = 1 \\ 2x & \text{if } x > 1 \end{cases}$

As, $f(1)$ is not defined.
Therefore, $f(x)$ is discontinuous at $x = 1$.

Q3:- If $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$

Discuss continuity at $x=2$ and $x=-2$.

Answer

At $x=2$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} (3)$$

$$= 3$$

L.H.L

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^2 - 1)$$

$$= (2)^2 - 1$$

$$= 4 - 1 = 3$$

R.H.L

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3)$$

$$= 3$$

As, $\lim_{x\to 2} f(x) = L.H.L = R.H.L$

Therefore, $f(x)$ is continuous at $x=2$.

Now, at $x=-2$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} (3x)$$

$$= 3(-2) = -6$$

L.H.L

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} (3x)$$

$$= 3(-2) = -6$$

R.H.L

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} (x^2-1)$$

$$= (-2)^2 - 1$$

$$= 4 - 1 = 3$$

As, $\lim_{x\to -2} f(x) = L.H.L \neq R.H.L$

Therefore, $f(x)$ is discontinuous at $x=-2$

Q4:- $f(x) = \begin{cases} x+2 & , x < -1 \\ c+2 & , x > -1 \end{cases}$

Find 'c' so that $\lim_{x\to -1} f(x)$ exists.

Answer

$\lim_{x\to -1} f(x) = \lim_{x\to -1} (x+2)$

$= -1+2 = 1$

L.H.L

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (x+2)$

$= -1+2 = 1$

R.H.L

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (c+2)$

$= c+2$

As, the limit exists at $c=-1$
Then

$$\lim_{x\to -1} f(x) = \lim_{x\to -1^+} f(x)$$

$$1 = c+2$$

$$1-2 = c$$

$$\boxed{-1 = c}$$

Q5:- Find the values of 'm' and 'n', so that $f(x)$ is continuous at $x=3$

i) $f(x) = \begin{cases} mx & \text{if } x<3 \\ n & \text{if } x=3 \\ -2x+9 & \text{if } x>3 \end{cases}$

Answer

$$\lim_{x\to 3} f(x) = \lim_{x\to 3}(n)$$

$$= n$$

L.H.L

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(mx)$$

$= m(3)$

$= 3m$

R.H.L

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (-2x+9)$$

$= -2(3)+9$

$= -6+9$

$= 3$

As, $f(x)$ is continuous at $x=3$.

Therefore,

$$\lim_{x \to 3} f(x) = L.H.L = R.H.L$$

$$n = 3m = 3$$

Hence,

$\boxed{n=3}$, $3m=3$

$m = \frac{3}{3}$

$\boxed{m=1}$

ii) $f(x)$ is continuous at $x=4$

$$f(x) = \begin{cases} mx & \text{if } x<4 \\ x^2 & \text{if } x \geq 4 \end{cases}$$

Answer

$$\lim_{x\to 4} f(x) = \lim_{x\to 4} (x^2)$$

$$= (4)^2$$

$$= 16$$

L.H.L

$$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (mx)$$

$$= m(4)$$

$$= 4m$$

As, $f(x)$ is continuous at $x=4$.

Therefore,

$$\lim_{x\to 4} f(x) = \text{L.H.L}$$

$$16 = 4m$$

$$\frac{16}{4} = m$$

$$\boxed{4 = m}$$

Q6:-

If $f(x) = \begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$

Find the value of k, so that f is continuous at $x=2$

Answer

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} k$$

$$= k$$

L.H.L

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}\left(\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}\right)$$

$$= \lim_{x\to 2^-}\left(\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \times \frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)$$

$$= \lim_{x\to 2^-}\left(\frac{(\sqrt{2x+5})^2-(\sqrt{x+7})^2}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}\right)$$

$$= \lim_{x\to 2^-} \frac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$= \lim_{x\to 2^-} \frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$$

$$= \lim_{x \to 2^-} \left(\frac{1}{\sqrt{2x+5} + \sqrt{x+7}} \right)$$

$$= \frac{1}{\sqrt{2(2)+5} + \sqrt{2+7}}$$

$$= \frac{1}{\sqrt{4+5} + \sqrt{9}}$$

$$= \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{3+3}$$

$$= \frac{1}{6}$$

As, $f(x)$ is continucous at $x=2$.

$$\lim_{x \to 2} f(x) = L.H.L$$

$$\boxed{k = \frac{1}{6}}$$