Exercise 5.3

Q1:- Maximize $f(x,y) = 2x + 5y$

subject to constraints

$2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$

Answer

$2y - x = 8$ —① ; $x - y = 4$ —②

For x-intercepts

Put $y = 0$ in eq ① & ②

$2(0) - x = 8$; $x - 0 = 4$

$-x = +8$; $x = 4$

$x = -8$;

Point is $(-8, 0)$; Point is $(4, 0)$

For y-intercepts

Put $x = 0$ in eq ① & ②

$2y - 0 = 8$; $0 - y = 4$

$2y = 8$; $-y = 4$

$y = 4$; $y = -4$

Point is $(0, 4)$; Point is $(0, -4)$

For intersection point

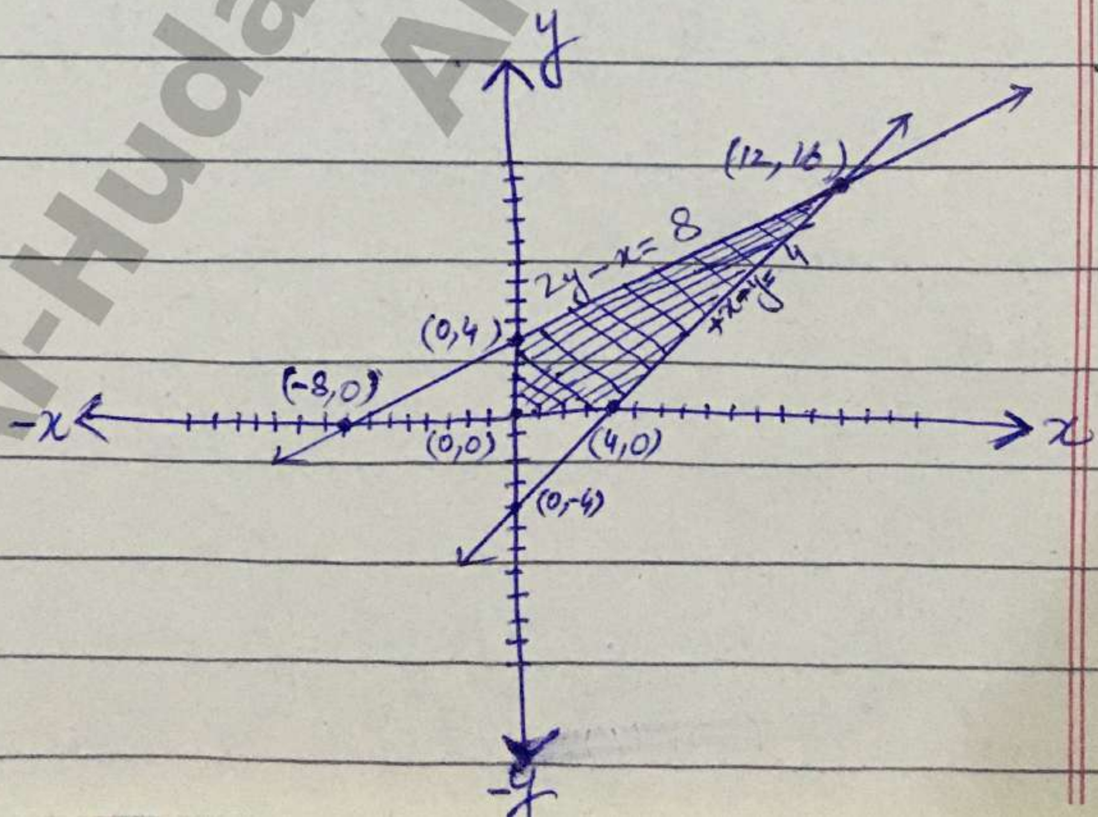
Adding ① & ②

$$-\cancel{x}+2y=8$$
$$\underline{\cancel{x}-y=4}$$
$$y=12$$

Put $y=12$ in ①

$$-x+2y=8$$
$$-x+2(12)=8$$
$$-x+24=8$$
$$-x=8-24$$
$$-x=-16$$
$$x=16$$

Point is $(16,12)$



Using Corner Points to find
$f(x, y) = 2x + 5y$

$f(4,0) = 2(4) + 5(0) = 8$
$f(0,0) = 2(0) + 5(0) = 0$
$f(0,4) = 2(0) + 5(4) = 20$
$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$

Hence,
function is maximum at the corner point $(16, 12)$

Q2:- Maximize $f(x,y) = x + 3y$
subject to constraints
$2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Answer

$2x + 5y \leq 30$ — ①; $5x + 4y \leq 20$ — ②

For x-intercepts

Put $y = 0$ in eq ① & ②

$2x + 5(0) = 30$; $5x + 4(0) = 20$

$2x = 30$; $5x = 20$

$x = 15$; $x = 4$

Point is $(15, 0)$; Point is $(4, 0)$

For y-intercepts

Put $x = 0$ in eq ① & ②

$2(0) + 5y = 30$; $5(0) + 4y = 20$

$5y = 30$; $4y = 20$

$y = 6$; $y = 5$

Point is $(0, 6)$; Point is $(0, 5)$

For intersection point

~~Add~~Using eq 1

$$2x+5y=30$$

$$2x=30-5y$$

$$x=\frac{30-5y}{2} \quad \text{---} \; 3$$

Put $x=\dfrac{30-5y}{2}$ in eq 2

$$5x+4y=20$$

$$5\left(\frac{30-5y}{2}\right)+4y=20$$

$$\frac{150-25y}{2}+4y=20$$

$$\frac{150-25y+8y}{2}=20$$

$$\frac{150-17y}{2}=20$$

$$75-\frac{17}{2}y=20$$

$$-\frac{17}{2}y=-55$$

$$y=$$

$$y = -55 \times \frac{-2}{17} = \frac{110}{17}$$

$$y = 6.5$$

Put $y = 6.5$ in eq (3)

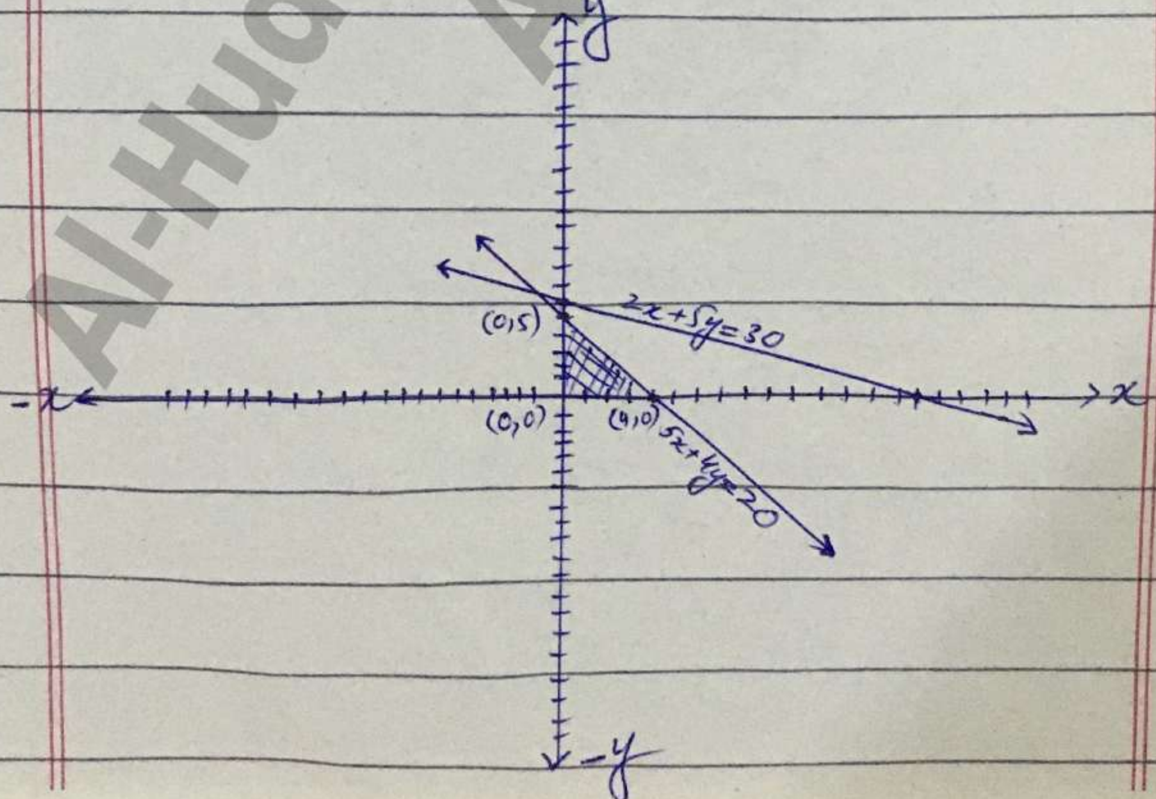
$$x = \frac{30-5y}{2}$$

$$x = \frac{30-5(6.5)}{2}$$

$$x = \frac{30-32.5}{2} = \frac{-2.5}{2}$$

$$x = -1.25$$

Point $(-1.25, 6.5)$ is not a corner point.



Using corner points to find

$f(x,y) = x+3y$

$f(0,0) = 0+3(0) = 0$

$f(4,0) = 4+3(0) = 4$

$f(0,5) = 0+3(5) = 15$

Hence,

function is maximum at the corner point $(0,5)$.

Q3:- Maximize $z = 2x+3y$

Subject to the constraints

$3x+4y \le 12$; $2x+y \le 4$; $4x-y \le 4$;

$x \ge 0$; $y \ge 0$

Answer

$3x+4y=12$ —① ; $2x+y=4$ —②; $4x-y=4$ —③

For x-intercepts

Put $y=0$ in eq ① & ② & ③

$3x+4(0)=12$; $2x+0=4$; $4x-0=4$

$3x=12$; $2x=4$; $4x=4$

$x=4$; $x=2$; $x=1$

Point is $(4,0)$; Point is $(2,0)$; Point is $(1,0)$

For y-intercepts

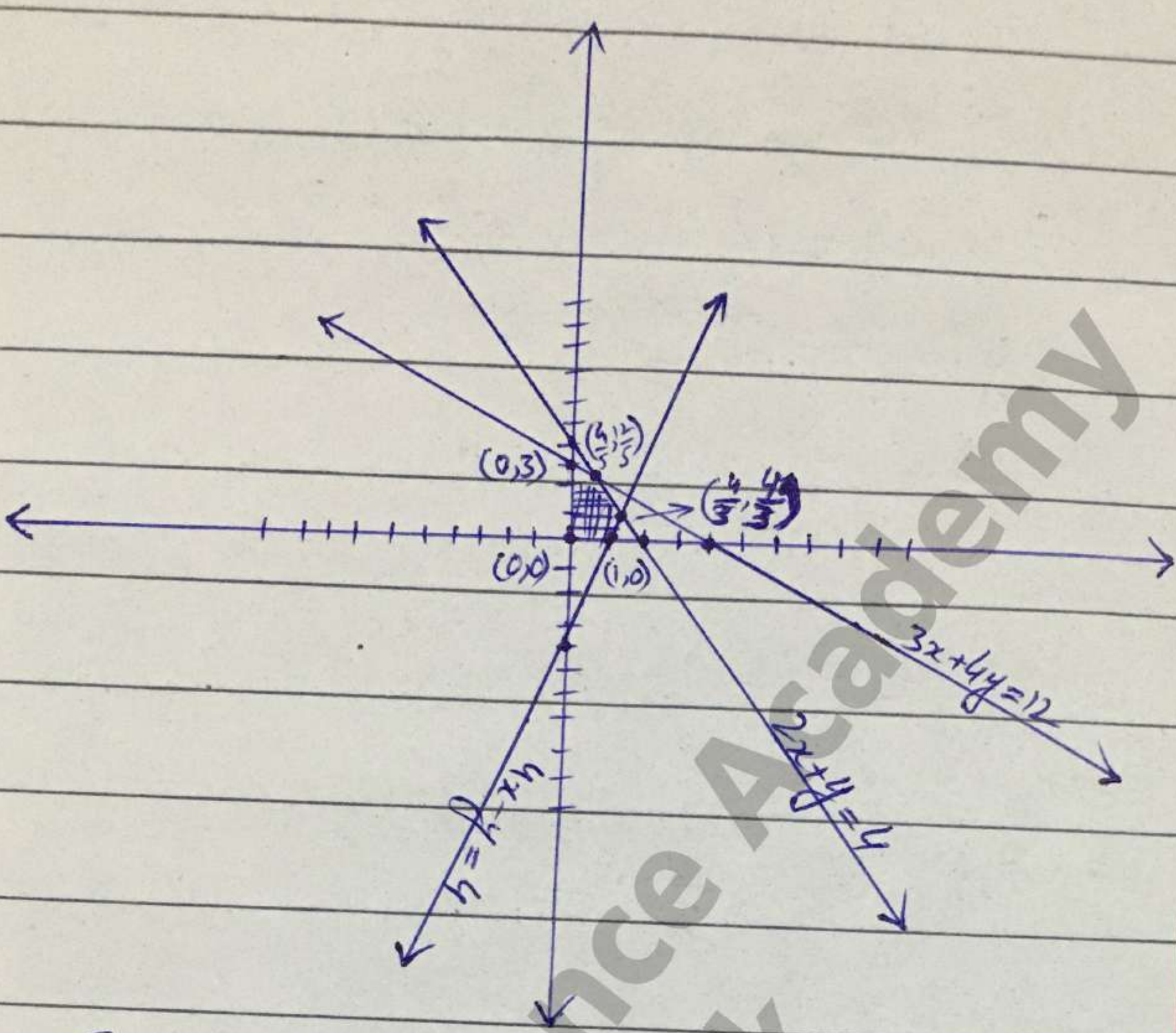
Put $x=0$ in eq ① & ② & ③

$3(0)+4y=12$; $2(0)+y=4$; $4(0)-y=4$

$4y=12$; $y=4$; $-y=4$

$y=3$; ; $y=-4$

Point is $(0,3)$; Point is $(0,4)$; Point is $(0,-4)$



For intersection point, using eq ① & ②

Using ②

$$2x+y=4$$

$$y=4-2x \quad \text{--- (3)}$$

Put $y=4-2x$ in ①

$$3x+4(4-2x)=12$$

$$3x+16-8x=12$$

$$-5x=12-16$$

$$-5x=-4$$

$$x=\frac{4}{5}$$

Put $x=\frac{4}{5}$ in ③

$$y=4-2x$$

$$y=4-2\left(\frac{4}{5}\right)$$

$$y=\frac{20-8}{5}$$

$$y=\frac{12}{5}$$

Point is $\left(\frac{4}{5},\frac{12}{5}\right)$.

For another intersection point using eq ② & ③

Adding ② & ③

$$2x+\cancel{y}=4$$

$$4x-\cancel{y}=4$$

$$6x=8$$

$$x=\frac{4}{3}$$

Put $x=\frac{4}{3}$ in ②

$$2x+y=4$$

$$2\left(\frac{4}{3}\right)+y=4$$

$$y = 4 - \frac{8}{3}$$

$$y = \frac{12-8}{3}$$

$$y = \frac{4}{3}$$

Point is $\left(\frac{4}{3}, \frac{4}{3}\right)$.

Using Corner Points to find

$z = 2x+3y$

$f(0,0) = 2(0)+3(0) = 0$

$f(1,0) = 2(1)+3(0) = 2$

$f\left(\frac{4}{3},\frac{4}{3}\right) = 2\left(\frac{4}{3}\right)+3\left(\frac{4}{3}\right) = \frac{8}{3}+\frac{12}{3} = \frac{20}{3}$

$f\left(\frac{4}{5},\frac{12}{5}\right) = 2\left(\frac{4}{5}\right)+3\left(\frac{12}{5}\right) = \frac{8}{5}+\frac{36}{5} = \frac{44}{5}$

$f(0,3) = 2(0)+3(3) = 9$

Hence,

function is maximum at corner point $(0,3)$

Q4:- Minimize $z = 2x + y$

Subject to constraints

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0, y \geq 0$

Answer

$x + y = 3$ —— ① ; $7x + 5y = 35$ —— ②

For x-intercepts

Put $y = 0$ in eq ① & ②

$x + 0 = 3$; $7x + 5(0) = 35$

$x = 3$; $7x = 35$

; $x = 5$

Point is $(3, 0)$; Point is $(5, 0)$

For y-intercepts

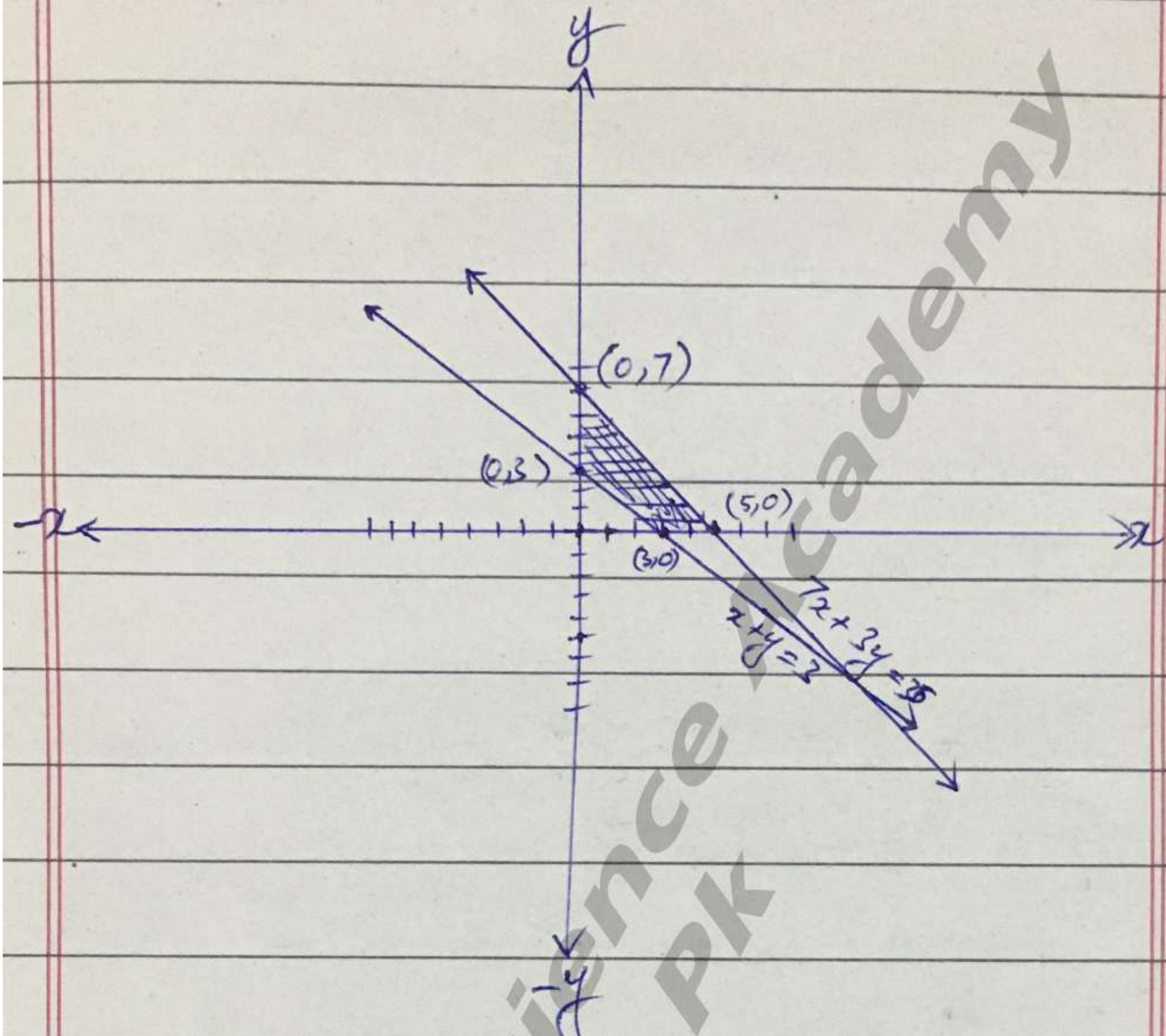
Put $x = 0$ in eq ① & ②

$0 + y = 3$; $7(0) + 5y = 35$

$y = 3$; $5y = 35$

; $y = 7$

Point is $(0, 3)$; Point is $(0, 7)$



Using Corner points to find

$z = 2x + y$

$f(3,0) = 2(3) + 0 = 6$

$f(5,0) = 2(5) + 0 = 10$

$f(0,3) = 2(0) + 3 = 3$

$f(0,7) = 2(0) + 7 = 7$

Hence,

function is minimum at corner point $(0,3)$

Q5:- Maximize the function defined as $f(x,y) = 2x+3y$ Subject to the constraints

$2x+y \leq 8$; $x+2y \leq 14$; $x \geq 0, y \geq 0$

<u>Answer</u>

$2x+y=8$ — ① ; $x+2y=14$ — ②

For <u>x-intercepts</u>

Put $y=0$ in eq ① & ②

$2x+0=8$; $x+2(0)=14$

$2x=8$; $x=14$

$x=4$;

Point is $(4,0)$; Point is $(14,0)$

For <u>y-intercepts</u>

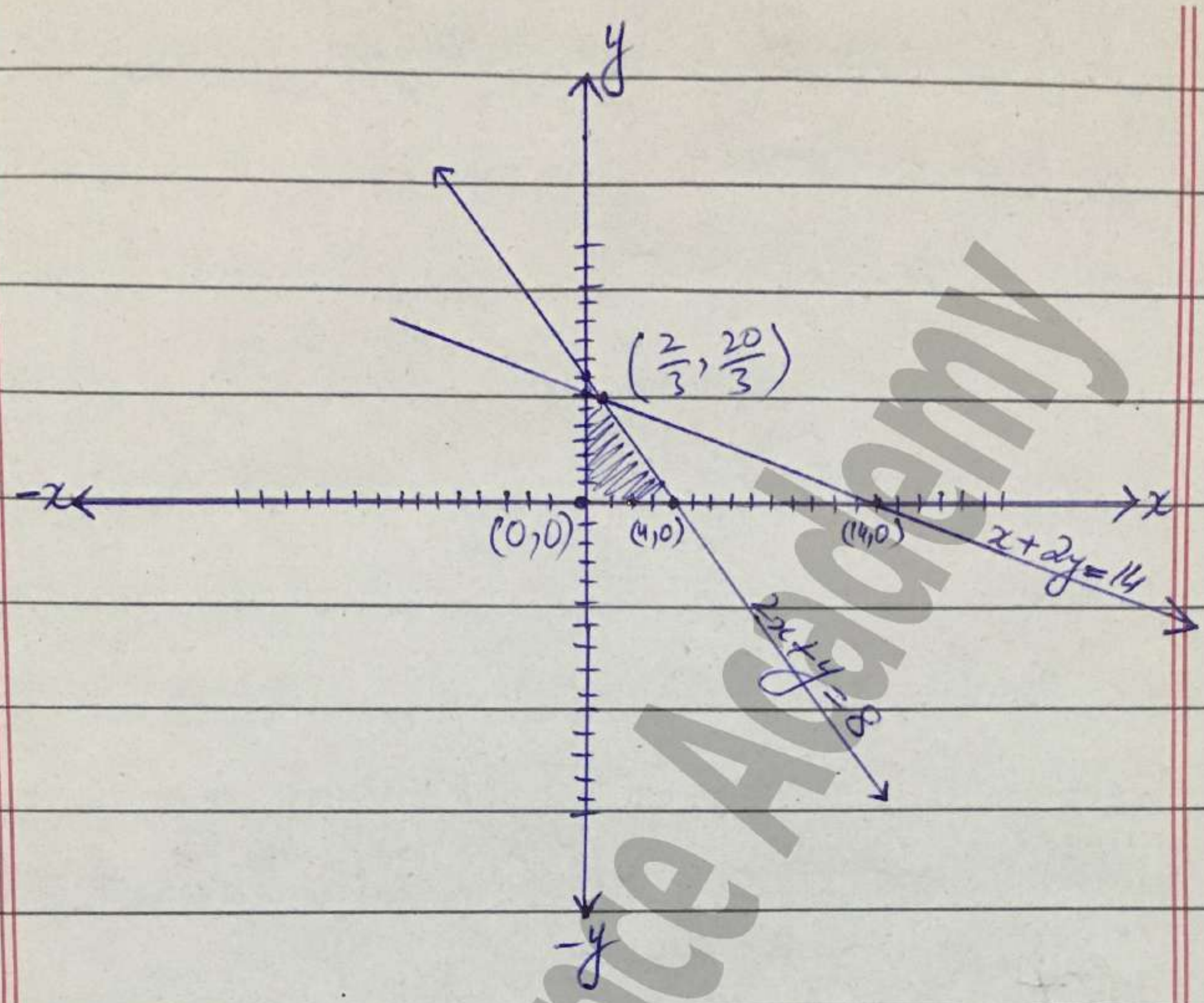
Put $x=0$ in eq ① & ②

$2(0)+y=8$; $0+2y=14$

$y=8$; $2y=14$

; $y=7$

Point is $(0,8)$; Point is $(0,7)$



For intersection point, using eq ① & ②

Using eq ①, $2x+y=8$

$$y=8-2x \quad \text{—③}$$

Put $y=8-2x$ in eq ②

$$x+2y=14$$
$$x+2(8-2x)=14$$
$$x+16-4x=14$$
$$-3x=14-16$$
$$-3x=-2$$
$$x=\frac{2}{3}$$

Put $x = \frac{2}{3}$ in eq ③

$$y = 8 - 2x$$

$$y = 8 - 2\left(\frac{2}{3}\right)$$

$$y = \frac{24-4}{3} = \frac{20}{3}$$

Point is $\left(\frac{2}{3}, \frac{20}{3}\right)$

Using Corner Points to find

$$f(x,y) = 2x + 3y$$

$$f(0,0) = 2(0) + 3(0) = 0$$

$$f(4,0) = 2(4) + 3(0) = 8$$

$$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{4+60}{3} = \frac{64}{3}$$

Hence,

function is maximum at corner point $\left(\frac{2}{3}, \frac{20}{3}\right)$.

Q6:- Minimize $z = 3x + y$

Subject to the constraints

$3x + 5y \geq 15$; $x + 6y \geq 9$; $x \geq 0$, $y \geq 0$

Answer

$3x + 5y = 15$ — ① ; $x + 6y = 9$ — ②

For x-intercepts

Put $y = 0$ in eq ① & ②

$3x + 5(0) = 15$; $x + 6(0) = 9$

$3x = 15$; $x = 9$

$x = 5$;

Point is $(5, 0)$; Point is $(9, 0)$

For y-intercepts

Put $x = 0$ in eq ① & ②

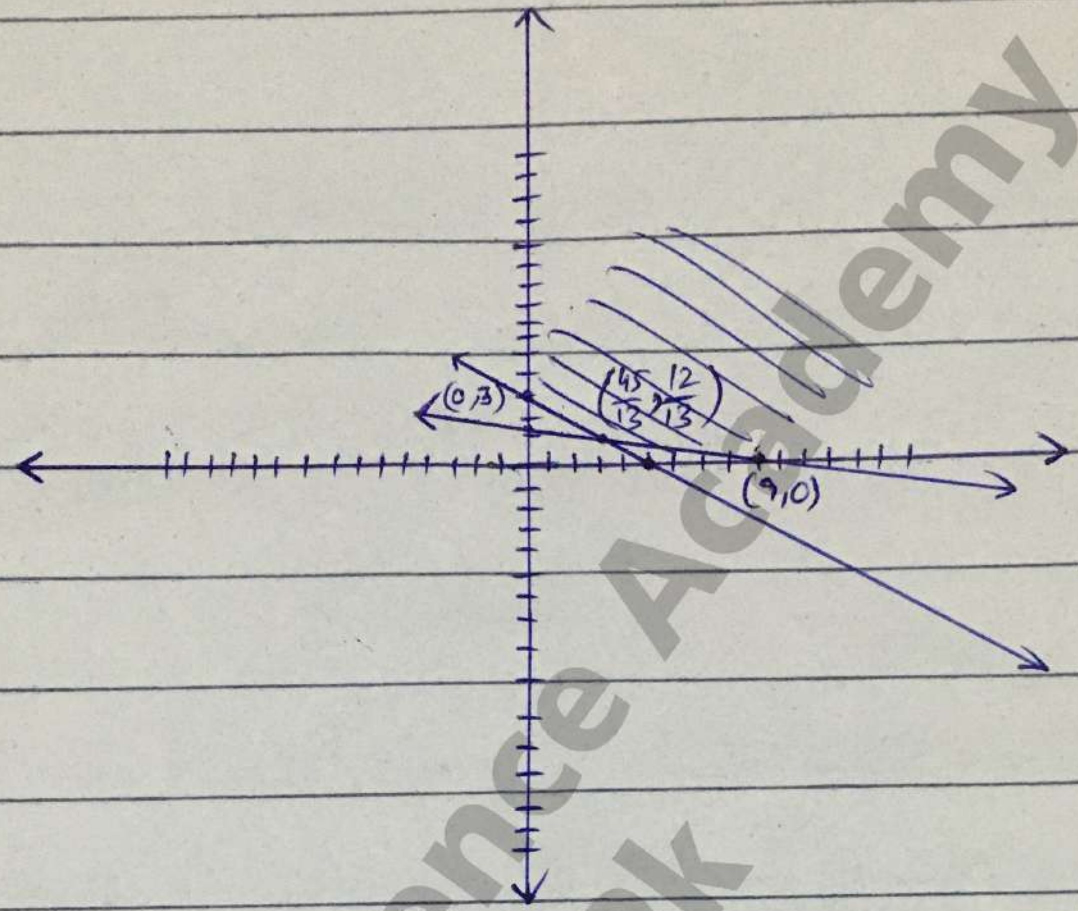
$3(0) + 5y = 15$; $0 + 6y = 9$

$5y = 15$; $6y = 9$

$y = 3$; $y = \frac{3}{2}$

;

Point is $(0, 3)$; Point is $(0, \frac{3}{2})$ $\rightarrow .5$



For intersection point, using eq ① & ②

Using eq ②, $x + 6y = 9$

$$x = 9 - 6y \quad \text{——} ③$$

Put $x = 9 - 6y$ in ①

$$3x + 5y = 15$$

$$3(9 - 6y) + 5y = 15$$

$$27 - 18y + 5y = 15$$

$$-13y = 15 - 27$$

$$-13y = -12$$

$$y = \frac{12}{13}$$

Put $y = \frac{12}{13}$ in eq (3)

$$x = 9 - 6y$$
$$x = 9 - 6\left(\frac{12}{13}\right)$$
$$x = \frac{117 - 72}{13}$$
$$x = \frac{45}{13}$$

Point is $\left(\frac{45}{13}, \frac{12}{13}\right)$

Using Corner Points to find

$$z = 3x + y$$
$$f(9,0) = 3(9) + 0 = 27$$
$$f(0,3) = 3(0) + 3 = 3$$
$$f\left(\frac{45}{13}, \frac{12}{13}\right) = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{135 + 12}{13} = \frac{147}{13}$$

Hence,

function is minimum at corner point $(0, 3)$