VECTORS

Torque and Equilibrium

$$\vec{A} = A\hat{A}$$

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$$\vec{A} = A\hat{A}$$

Introduction:

a. Unit vector:

The unit vector is used to represent the direction of the vector.

A unit vector is given by,

$$\hat{A} = \frac{\vec{A}}{A}$$

b. Rectangular components of vector:

A vector can dissolve into two components which are directed perpendicular to each other. Such components are called Rectangular components of a vector. They are dissolved along x-axis and y-axis and are given by,

Magnitude of $A_x\hat{\boldsymbol{\imath}}$ or x-component of $\vec{A}$ = $A\cos\theta$.

And

Magnitude of $A_y\hat{\boldsymbol{\jmath}}$ or y-component of $\vec{A}$ = $A\sin\theta$

c. Determination of vector from its rectangular components

If rectangular components are given, then we determine the vector:

$$A = \sqrt{{A_x}^2 + {A_y}^2}$$

And angle is given by,

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

d. Position vector

It is a vector which describes the location of some points with respect to their origin.

$$\boldsymbol{r} = a\hat{\boldsymbol{\imath}} + b\hat{\boldsymbol{\jmath}}$$

Then $$r = \sqrt{a^2 + b^2}$$

e. Unit vector

It is a vector which has magnitude one and it is used to describe the direction of a given vector. e.g. $\hat{\boldsymbol{\imath}}$ is a unit vector along x-axis and $\hat{\boldsymbol{\jmath}}$ is a unit vector along y-axis and $\hat{\boldsymbol{k}}$ is unit vector along z-axis.

Vector addition by rectangular components

The vector addition by rectangular components involves the following steps:

1. Find x any y components of all the vectors.
2. Find x-component R_x of the resultant vector by adding the x-components.
3. Find y-component R_y of the resultant vector by adding y-components.
4. Find the magnitude R of the resultant vector by the following formula,

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

5. The angle θ of the resultant vector is given by,

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Note:

- If R_x *and* R_y are positive then the angel is written as it.
- If R_x *is negative and* R_y is positive then they lie in second quadrant and angle is given by; $\theta = 180 - \phi$
- If R_x and R_y both are negative then the resultant vector lie in third quadrant and angle is given by; $\theta = 180 + \phi$
- If R_x is positive and R_y is negative then the resultant vector lie in fourth quadrant and angle is given by; $\theta = 360 - \phi$

Scalar or Dot product

The scalar product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ is given by,

$$\boldsymbol{A}.\boldsymbol{B} = AB\cos\theta$$

Where θ is angle between $\boldsymbol{A}$ and $\boldsymbol{B}$.

Characteristics of Scalar product:

- The scalar product is commutative.

i.e. $$\boldsymbol{A}.\boldsymbol{B} = \boldsymbol{B}.\boldsymbol{A}$$

- Their product is zero when angle between $\boldsymbol{A}$ and $\boldsymbol{B}$ is $90°$.

i.e. $$\boldsymbol{A}.\boldsymbol{B} = AB\cos 90° = 0$$

- The product of unit vector along x, y, and z-axis is also zero.

i.e. $$\hat{\boldsymbol{\imath}}.\hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\jmath}}.\hat{\boldsymbol{k}} = \hat{\boldsymbol{k}}.\hat{\boldsymbol{\imath}} = \boldsymbol{0}$$

- The scalar product of vector with itself is equal to the square of its magnitude.

i.e. $$\boldsymbol{A}.\boldsymbol{A} = (A)^2\cos 0° = A^2$$

- The scalar product of unit vector with itself is equal to 1

i.e. $$\hat{\boldsymbol{\imath}}.\hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}}.\hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}}.\hat{\boldsymbol{k}} = 1$$

- The scalar product of two parallel vectors is equal to the product of their magnitude.

i.e. $$\boldsymbol{A}.\boldsymbol{B} = AB\cos 0° = AB$$

- Scalar product of two vectors in terms of their rectangular components:

$$\boldsymbol{A}.\boldsymbol{B} = A_x.B_x + A_yB_y + A_zB_z$$

- The angle between two vectors can be find:

$$\cos\theta = \frac{A_x.B_x + A_yB_y + A_zB_z}{AB}$$

Vector or Cross product

The vector or cross product of two vectors is defined by:

$$\boldsymbol{A} \times \boldsymbol{B} = AB\sin\theta\,\hat{n}$$

Where $\hat{n}$ is a unit vector and it is perpendicular to the plane containing $\boldsymbol{A}$ *and* $\boldsymbol{B}$.

Characteristics of vector product:

- The cross product is not commutative,

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

- The cross product of two parallel vectors or anti-parallel vectors is null vector.

i.e. $$\boldsymbol{A} \times \boldsymbol{B} = AB \sin 0^\circ \hat{n} = \boldsymbol{0}$$

- The cross product of unit vector with itself is also zero.

$$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{k}} = 0$$

Also $$\boldsymbol{A} \times \boldsymbol{A} = \boldsymbol{0}$$

- The cross product of two mutually perpendicular vectors has maximum value which is given by:

$$\boldsymbol{A} \times \boldsymbol{B} = AB\hat{n}$$

- The cross product of unit vectors along x-axis, y-axis and z-axis is given by:

$$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}}$$

$$\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}$$

$$\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}$$

- Cross product of two vectors in terms of its rectangular components is given by,

$$\boldsymbol{A} \times \boldsymbol{B} = \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Torque

Torque is given by,

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

$\Rightarrow$ $$\tau = rF \sin\theta \, \hat{n}$$

Where θ is angle between $\boldsymbol{r}$ and $\boldsymbol{F}$.

$\boldsymbol{r}$ is position vector of moment arm. Moment arm is the perpendicular distance between pivot point and the line of the action of the force. Torque is a vector quantity and its SI unit is Nm. Torque is similar with force in rotational motion as force in linear motion.

Equilibrium

A body is said to be in equilibrium when it is at rest or moving with uniform velocity.

First condition of equilibrium:

When the sum of all the forces acting on the body is zero then first condition of equilibrium is satisfied.

i.e. $$\Sigma \boldsymbol{F} = 0$$

Second condition of equilibrium

When the sum of all the torques acting on the body is zero then the second condition is satisfied.

$$\Sigma \tau = 0$$

When the first condition is satisfied then there is no linear acceleration and body will be in translational equilibrium.

When the second condition of equilibrium is satisfied then there is no angular acceleration and body will be in rotational equilibrium.

If a body is at rest, it is said to be in static equilibrium and when body moves with uniform velocity then it is said to be in dynamic equilibrium.