

Ex # 10.2

Q1:-

$$i) \sin(180^\circ + \theta) = -\sin \theta$$

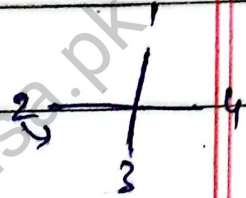
L.H.S.

$$= \sin(180^\circ + \theta)$$

$$= \sin(2 \times 90^\circ + \theta)$$

$$= -\sin \theta$$

= R.H.S.



$$ii) \tan(270^\circ - \theta) = \cot \theta$$

L.H.S.

$$= \tan(270^\circ - \theta)$$

$$= \tan(3 \times 90^\circ - \theta)$$

$$= \cot \theta$$

= R.H.S.

$$iii) \cos(\theta - 180^\circ) = -\cos \theta$$

L.H.S.

$$= \cos(180^\circ - \theta)$$

$$= \cos(2 \times 90^\circ - \theta)$$

$$\because \cos(-\theta) = \cos \theta$$

$$= -\cos \theta$$

= R.H.S.

~~*~~

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Mon Tue Wed Thu Fri Sat

Page No.

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Q.2

$$i) \sin 15^\circ$$

$$= \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$ii) \cos 15^\circ$$

$$= \cos(45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Mon Tue Wed Thu Fri Sat

Date: ___/___/___

$$\text{ex) } \tan 5^\circ$$

$$= \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

Q.2-

$$\begin{aligned} \text{iv) } \sin 105^\circ &= \sin(60^\circ + 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{v) } \cos(105^\circ) &= \cos(60^\circ + 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned}
 \text{vi)} \quad & \tan 105^\circ \\
 &= \tan (60^\circ + 45^\circ) \\
 &= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} \\
 &= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1} \\
 &= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Q. 2:-} \quad & \text{L.H.S} \\
 & \sin (45^\circ + x) \\
 &= \sin 45^\circ \cos x + \cos 45^\circ \sin x \\
 &= \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \\
 &= \frac{1}{\sqrt{2}} (\cos x + \sin x) \\
 &= \frac{1}{\sqrt{2}} (\sin x + \cos x) \\
 &= \text{R.H.S}
 \end{aligned}$$

ii) L.H.S.

$$= \tan\left(\frac{\pi}{4} - \theta\right) + \tan\left(\frac{3\pi}{4} + \theta\right)$$

$$\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}$$

$$+ \frac{\tan\frac{3\pi}{4} + \tan\theta}{1 - \tan\frac{3\pi}{4}\tan\theta}$$

$$= \frac{(1) - \tan\theta}{1 + (1)\tan\theta} + \frac{-1 + \tan\theta}{1 - (-1)\tan\theta}$$

$$= \frac{1 - \tan\theta}{1 + \tan\theta} + \frac{-1 + \tan\theta}{1 + \tan\theta}$$

$$= \frac{1 - \tan\theta - 1 + \tan\theta}{1 + \tan\theta}$$

$$= \frac{0}{1 + \tan\theta}$$

$$= 0$$

$$= \text{R.H.S.}$$

iii) L.H.S.

$$= \sin\left(0 + \frac{\pi}{6}\right) + \cos\left(0 + \frac{\pi}{3}\right)$$

$$= \frac{\sin 0 \cos \frac{\pi}{6} + \cos 0 \sin \frac{\pi}{6}}{6} + \frac{\cos 0 \cos \frac{\pi}{3}}{3}$$

$$- \frac{\sin 0 \sin \frac{\pi}{3}}{3}$$

$$= \frac{\cancel{\sin 0} \times \frac{\sqrt{3}}{2}}{2} + \cos 0 \times \frac{1}{2} + \cos 0 \times \frac{1}{2} - \frac{\cancel{\sin 0} \times \frac{\sqrt{3}}{2}}{2}$$

$$= \frac{2 \cos 0}{2}$$

$$= \cos 0$$

$$= R.H.S.$$

Q. 2

iv) L.H.S.

$$= \frac{\sin \theta - \cos \theta \tan \frac{\theta}{2}}{\cos \theta + \sin \theta \tan \frac{\theta}{2}}$$

$$= \frac{\sin \theta - \cos \theta \frac{\sin \theta / 2}{\cos \theta / 2}}{\cos \theta + \sin \theta \frac{\sin \theta / 2}{\cos \theta / 2}}$$

$$= \frac{\sin \theta - \cos \theta \frac{\sin \theta / 2}{\cos \theta / 2}}{\cos \theta + \sin \theta \frac{\sin \theta / 2}{\cos \theta / 2}}$$

$$= \frac{\sin \theta - \cos \theta \frac{\sin \theta / 2}{\cos \theta / 2}}{\cos \theta + \sin \theta \frac{\sin \theta / 2}{\cos \theta / 2}}$$

$$\frac{\sin \theta \cos \theta/2 - \cos \theta \sin \theta/2}{\cos \theta/2}$$

$$\frac{\cos \theta \cos \theta/2 + \sin \theta \sin \theta/2}{\cos \theta/2}$$

$$\frac{\sin(\theta - \frac{\theta}{2})}{\cos(\theta - \frac{\theta}{2})}$$

$$\frac{\sin(\frac{2\theta - \theta}{2})}{\cos(\frac{2\theta - \theta}{2})}$$

$$\frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

$$= \tan \frac{\theta}{2}$$

= R.H.S.

✓)

L.H.S.

$$= \frac{1 - \tan \theta \tan \phi}{1 + \tan \theta \tan \phi}$$

$$= \frac{1 - \frac{\sin \theta}{\cos \theta} \times \frac{\sin \phi}{\cos \phi}}{1 + \frac{\sin \theta}{\cos \theta} \times \frac{\sin \phi}{\cos \phi}}$$

$$= \frac{1 - \frac{\sin \theta}{\cos \theta} \times \frac{\sin \phi}{\cos \phi}}{1 + \frac{\sin \theta}{\cos \theta} \times \frac{\sin \phi}{\cos \phi}}$$

$$= \frac{\cos \theta \cos \phi - \sin \theta \sin \phi}{\cos \theta \cos \phi}$$

$$= \frac{\cos \theta \cos \phi + \sin \theta \sin \phi}{\cos \theta \cos \phi}$$

$$= \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)}$$

$$= \frac{\cos(\theta + \phi)}{\cos(\theta - \phi)}$$

$$= R.H.S.$$

L.H.S.

$$\text{Q5e- } \cos(\alpha + \beta) \cos(\alpha - \beta)$$

$$= (\cos \alpha \cos \beta - \sin \alpha \sin \beta)(\cos \alpha \cos \beta + \sin \alpha \sin \beta)$$

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta$$

$$= \cos^2 \alpha (1 - \sin^2 \beta) - (1 - \cos^2 \alpha) \sin^2 \beta$$

$$= \cos^2 \alpha - \cos^2 \alpha \sin^2 \beta - \sin^2 \beta + \cos^2 \alpha \sin^2 \beta$$

$$= \cos^2 \alpha - \sin^2 \beta \quad \text{Proved}$$

Now, using

$$= \cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta$$

$$= (1 - \sin^2 \alpha) \cos^2 \beta - \sin^2 \alpha (1 - \cos^2 \beta)$$

$$= \cos^2 \beta - \sin^2 \alpha \cos^2 \beta - \sin^2 \alpha + \sin^2 \alpha \cos^2 \beta$$

$$= \cos^2 \beta - \sin^2 \alpha \quad \text{Proved}$$

Q.6 C.H.S.

$$= \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{\cos(\alpha+\beta) + \cos(\alpha-\beta)}$$

$$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

$$= \frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= \text{R.H.S.}$$

Q7- i)

R.H.S.

$$= \frac{\cot \alpha \cot \beta - 1}{\cot \alpha + \cot \beta}$$

$$\frac{1}{\tan \alpha} \times \frac{1}{\tan \beta} - 1$$

$$= \frac{1}{\tan \alpha} + \frac{1}{\tan \beta}$$

$$\frac{1 - \tan \alpha \tan \beta}{\tan \alpha \tan \beta}$$

$$\frac{\tan \beta + \tan \alpha}{\tan \alpha \tan \beta}$$

$$= \frac{1 - \tan \alpha \tan \beta}{\tan \alpha + \tan \beta} = \frac{1}{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}}$$

$$= \frac{1}{\tan(\alpha + \beta)}$$

$$= \cot(\alpha + \beta)$$

= L.H.S.

$$\therefore \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

ii) R.H.S.

$$= \frac{\cot \alpha \cot \beta + 1}{\cot \beta - \cot \alpha}$$

$$= \frac{1}{\tan \alpha} \times \frac{1}{\tan \beta} + 1$$

$$= \frac{\frac{1}{\tan \beta} - \frac{1}{\tan \alpha}}{1 + \frac{1}{\tan \alpha} \times \frac{1}{\tan \beta}}$$

$$= \frac{\frac{1}{\tan \beta} - \frac{1}{\tan \alpha}}{1 + \frac{1}{\tan \alpha \tan \beta}}$$

$$= \frac{\frac{\tan \alpha - \tan \beta}{\tan \alpha \tan \beta}}{1 + \frac{1}{\tan \alpha \tan \beta}}$$

$$= \frac{\tan \alpha - \tan \beta}{\tan \alpha \tan \beta + 1}$$

$$= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$= \frac{1}{\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}}$$

$$= \frac{1}{\tan(\alpha - \beta)}$$

$$= \cot(\alpha - \beta)$$

$$= \text{L.H.S.}$$

iii)

L.H.S.

$$= \frac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{\frac{\sin \alpha}{\cos \alpha} - \frac{\sin \beta}{\cos \beta}}$$

$$= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta}$$

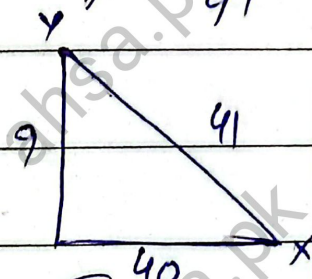
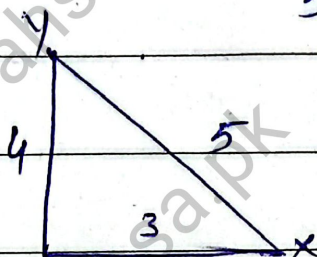
$$= \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)}$$

$$= \text{R.H.S.}$$

$$\text{Hence,}$$

$$\text{Q.E.D.}$$

Q8:- $\sin \alpha = \frac{4}{5}$, $\cos \beta = \frac{40}{41}$



Using Pythagoras theorem

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2 ; (\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$(5)^2 = (\text{base})^2 + (4)^2 ; (41)^2 = (40)^2 + (\text{perp})^2$$

$$25 - 16 = (\text{base})^2 ; 1681 - 1600 = (\text{perp})^2$$

$$9 = (\text{base})^2 ; 81 = (\text{perp})^2$$

$$3 = \text{base} ; 9 = \text{perp}$$

$$\cos \alpha = \frac{B}{H} = \frac{3}{5} ; \sin \beta = \frac{P}{H} = \frac{9}{41}$$

$$\sin(\alpha - \beta) = \frac{133}{205}$$

L.H.S.

$$= \sin(\alpha - \beta)$$

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$= \frac{4}{5} \times \frac{40}{41} - \frac{3}{5} \times \frac{9}{41}$$

$$= \frac{160}{205} - \frac{27}{205}$$

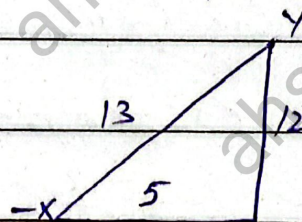
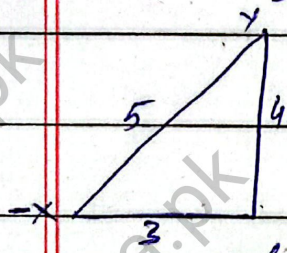
$$= \frac{133}{205}$$

$$= R.H.S.$$

Q9:-

$$\sin \alpha = \frac{4}{5}$$

$$\sin \beta = \frac{12}{13}$$



Using Pythagoras Theorem

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2, \quad (\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$(5)^2 = (\text{base})^2 + (4)^2, \quad (13)^2 = (\text{base})^2 + (12)^2$$

$$25 - 16 = (\text{base})^2, \quad 169 - 144 = (\text{base})^2$$

$$9 = (\text{base})^2, \quad 25 = (\text{base})^2$$

$$3 = \text{base}, \quad 5 = \text{base}$$

$$\cos \alpha = \frac{B}{H} = \frac{-3}{5}$$

$$\cos \beta = \frac{B}{H} = \frac{-5}{13}$$

$$\tan \alpha = \frac{P}{B} = \frac{-4}{3}$$

$$\tan \beta = \frac{P}{B} = \frac{-12}{5}$$

$$i) \sin(\alpha + \beta)$$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{4}{5} \times \left(\frac{-5}{13}\right) + \left(\frac{-3}{5}\right) \times \frac{12}{13}$$

$$= \frac{-20}{65} - \frac{36}{65}$$

$$= \frac{-56}{65}$$

$$ii) \cos(\alpha + \beta)$$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(\frac{-3}{5}\right) \times \left(\frac{-5}{13}\right) - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{-33}{65}$$

$$iii) \tan(\alpha + \beta)$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{-\frac{4}{3} + \left(\frac{-12}{5}\right)}{1 - \left(\frac{-4}{3}\right)\left(\frac{-12}{5}\right)}$$

$$= \frac{-\frac{4}{3} - \frac{12}{5}}{1 - \frac{48}{15}}$$

$$= \frac{\frac{-20 - 36}{15}}{\frac{15 - 48}{15}}$$

$$= \frac{+56}{+33}$$

$$= \frac{56}{33}$$

$$ii) \sin(\alpha - \beta)$$

$$= \sin\alpha \cos\beta - \cos\alpha \sin\beta$$

$$= \frac{4}{5} \times \left(\frac{-5}{13}\right) - \left(\frac{-3}{5}\right) \times \frac{12}{13}$$

$$= \frac{-20}{65} + \frac{36}{65}$$

$$= \frac{16}{65}$$

$$iii) \cos(\alpha - \beta)$$

$$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$$

$$= \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

$$= \frac{+15}{65} + \frac{48}{65} = \frac{63}{65}$$

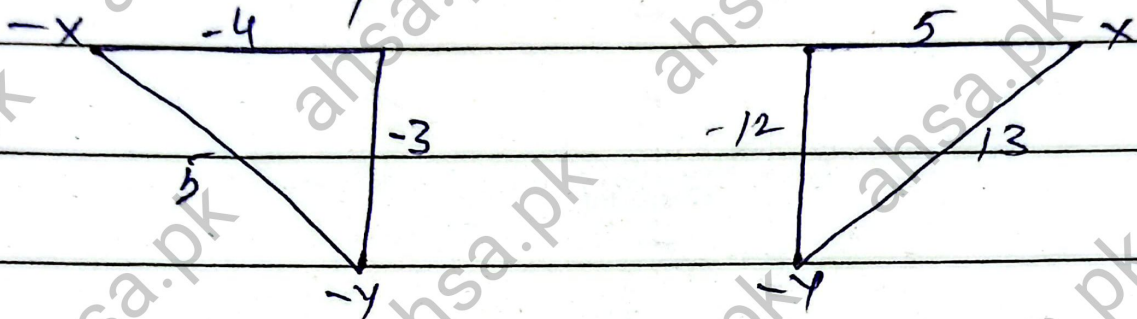
$$\begin{aligned}
 \text{ve)} \quad & \tan(\alpha - \beta) \\
 &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\
 &= \frac{-\frac{4}{3} - \left(-\frac{12}{5}\right)}{1 + \frac{-4}{3} \times \left(-\frac{12}{5}\right)}
 \end{aligned}$$

$$= \frac{\frac{-20 + 48}{15}}{\frac{15 + 48}{15}}$$

$$= \frac{16}{63}$$

Q.10:- $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$

i) $\tan \alpha = \frac{3}{4}$, $\cos \beta = \frac{5}{13}$



Using Pythagoras Theorem

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2, \quad (\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$(\text{hyp})^2 = (-4)^2 + (-3)^2, \quad (13)^2 = (5)^2 + (\text{perp})^2$$

$$(\text{hyp})^2 = 16 + 9, \quad 169 - 25 = (\text{perp})^2$$

$$(\text{hyp})^2 = 25, \quad 144 = (\text{perp})^2$$

$$\text{hyp} = 5, \quad 12 = \text{perp}$$

$$\sin \alpha = \frac{P}{H} = \frac{-3}{5}, \quad \sin \beta = \frac{P}{H} = \frac{-12}{13}$$

$$\cos \alpha = \frac{B}{H} = \frac{-4}{5}$$

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$= \frac{-3}{5} \times \frac{5}{13} + \frac{-4}{5} \times \frac{-12}{13}$$

$$= \frac{-15}{65} + \frac{48}{65}$$

$$= \frac{33}{65}$$

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

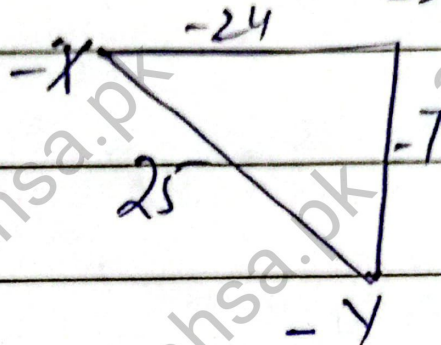
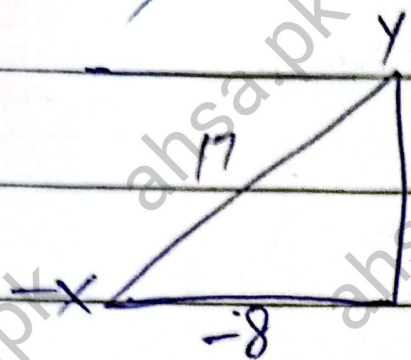
$$= \frac{-4}{5} \times \frac{5}{13} - \frac{-3}{5} \times \frac{-12}{13}$$

$$= \frac{-20}{65} - \frac{36}{65}$$

$$= \frac{-56}{65}$$

Q. 100 (ii) $\tan \alpha = -\frac{15}{8}$

$\tan \sin \beta = -\frac{7}{25}$



Using Pythagoras Theorem.

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$(\text{hyp})^2 = (-8)^2 + (15)^2$$

$$(25)^2 = (\text{base})^2 + (-7)^2$$

$$(\text{hyp})^2 = 64 + 225$$

$$625 = (\text{base})^2 + 49$$

$$(\text{hyp})^2 = 289$$

$$625 - 49 = (\text{base})^2$$

$$\sqrt{(\text{hyp})^2} = \sqrt{17^2}$$

$$576 = (\text{base})^2$$

$$\text{hyp} = 17$$

$$24 = \text{base}$$

$$\sin \alpha = \frac{P}{H} = \frac{15}{17}$$

$$\cos \beta = \frac{B}{H}$$

$$\cos \alpha = \frac{B}{H} = \frac{-8}{17}$$

$$\cos \beta = \frac{-24}{25}$$

$$\sin(\alpha + \beta)$$

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \frac{15}{17} \times \frac{-24}{25} + \frac{-8}{17} \times \frac{-7}{25}$$

$$= \frac{-360}{425} + \frac{56}{425}$$

$$= \frac{-304}{425}$$

$$\cos(\alpha + \beta)$$

$$= \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \frac{-8}{17} \times \frac{-24}{25} - \frac{15}{17} \times \frac{-7}{25}$$

$$= \frac{192}{425} + \frac{105}{425}$$

$$= \frac{297}{425}$$

Q.11:- R.H.S.

$$= \tan 37^\circ$$

$$= \tan (45^\circ - 8^\circ)$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \tan 8^\circ}$$

$$= \frac{1 - \tan 8^\circ}{1 + (1) \tan 8^\circ}$$

$$= \frac{1 - \frac{\sin 8^\circ}{\cos 8^\circ}}{1 + \frac{\sin 8^\circ}{\cos 8^\circ}}$$

$$= \frac{1 - \frac{\sin 8^\circ}{\cos 8^\circ}}{1 + \frac{\sin 8^\circ}{\cos 8^\circ}}$$

$$= \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$= \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$= \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$= \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$= \text{L.H.S}$$

Q12:-

As,

$$\alpha + \beta + \gamma = 180^\circ$$

$$\alpha + \beta = 180^\circ - \gamma$$

$$\frac{\alpha + \beta}{2} = \frac{180^\circ - \gamma}{2}$$

$$\frac{\alpha}{2} + \frac{\beta}{2} = 90^\circ - \frac{\gamma}{2}$$

Taking 'tan' on both sides.

$$\tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(90^\circ - \frac{\gamma}{2}\right)$$

$$\tan \frac{\alpha}{2} + \tan \frac{\beta}{2}$$

$$= \cot \frac{\gamma}{2}$$

$$1 - \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$

$$\frac{1}{\cot \frac{\alpha}{2}} + \frac{1}{\cot \frac{\beta}{2}}$$

$$= \cot \frac{\gamma}{2}$$

$$1 - \frac{1}{\cot \frac{\alpha}{2} \cot \frac{\beta}{2}}$$

$$\frac{\cot \frac{\beta}{2} + \cot \frac{\alpha}{2}}{\cot \frac{\alpha}{2} \cot \frac{\beta}{2}}$$

$$= \cot \frac{\gamma}{2}$$

$$\frac{\cot \frac{\alpha}{2} \cot \frac{\beta}{2} - 1}{\cot \frac{\alpha}{2} \cot \frac{\beta}{2}}$$

$$= \cot \frac{\gamma}{2}$$

$$\frac{\cot \alpha/2 + \cot \beta/2}{\cot \alpha/2 \cot \beta/2 - 1} = \cot \frac{\gamma}{2}$$

$$\cot \alpha/2 + \cot \beta/2 = \cot \frac{\gamma}{2} (\cot \alpha/2 \cot \beta/2 - 1)$$

$$\cot \alpha/2 + \cot \beta/2 = \cot \alpha/2 \cot \beta/2 \cot \frac{\gamma}{2} - \cot \frac{\gamma}{2}$$

$$\cot \alpha/2 + \cot \beta/2 + \cot \frac{\gamma}{2} = \cot \alpha/2 \cot \beta/2 \cot \frac{\gamma}{2}$$

Hence Proved

Q13:-

$$\text{As, } \alpha + \beta + \gamma = 180^\circ$$

$$\alpha + \beta = 180^\circ - \gamma$$

Taking 'tan' on both sides-

$$\tan(\alpha + \beta) = \tan(2 \times 90^\circ - \gamma)$$

$$\tan(\alpha + \beta) = -\tan \gamma$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = -\tan \gamma$$

$$\frac{\frac{1}{\cot \alpha} + \frac{1}{\cot \beta}}{1 - \frac{1}{\cot \alpha} \times \frac{1}{\cot \beta}} = -\frac{1}{\cot \gamma}$$

$$\frac{\cot \beta + \cot \alpha}{\cot \alpha \cot \beta} = -\frac{1}{\cot \gamma}$$

$$\frac{\cot \alpha \cot \beta - 1}{\cot \alpha \cot \beta}$$

$$\frac{\cot \alpha + \cot \beta}{\cot \alpha \cot \beta - 1} = -\frac{1}{\cot \gamma}$$

$$(\cot \alpha + \cot \beta) \cot \gamma = -1(\cot \alpha \cot \beta - 1)$$

$$\cot \gamma \cot \alpha + \cot \beta \cot \gamma = -\cot \alpha \cot \beta + 1$$

$$\cot \alpha \cot \beta + \cot \beta \cot \gamma + \cot \gamma \cot \alpha = 1$$

Hence Proved

Q14:-

i) $12\sin\theta + 5\cos\theta$

Using Formula

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Let

$$12 = r\cos\phi \text{ --- (1) and } 5 = r\sin\phi \text{ --- (2)}$$

Squaring and Adding (1) & (2)

$$12^2 + 5^2 = r^2\cos^2\phi + r^2\sin^2\phi$$

$$144 + 25 = r^2(\cos^2\phi + \sin^2\phi)$$

$$169 = r^2(1)$$

$$\sqrt{169} = \sqrt{r^2}$$

$$13 = r$$

Dividing (1) & (2)

$$\frac{5}{12} = \frac{r\sin\phi}{r\cos\phi}$$

$$\frac{5}{12} = \tan\phi$$

$$\tan^{-1} \frac{5}{12} = \phi$$

Then, given equation becomes

$$= 2 \cos \phi \sin \theta + 2 \sin \phi \cos \theta$$

$$= 2 (\sin \theta \cos \phi + \cos \theta \sin \phi)$$

$$= 13 \sin(\theta + \phi)$$

$$= 13 \sin\left(\theta + \tan^{-1} \frac{5}{12}\right) \quad \therefore \phi = \tan^{-1} \frac{5}{12}$$

ii) $3 \sin \theta - 4 \cos \theta$

Using Formula

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Let

$$3 = 2 \cos \phi \quad \text{--- (1)} \quad 4 = 2 \sin \phi \quad \text{--- (2)}$$

Adding & Squaring (1) & (2)

$$3^2 + 4^2 = 2^2 \cos^2 \phi + 2^2 \sin^2 \phi$$

$$9 + 16 = 2^2 (\cos^2 \phi + \sin^2 \phi)$$

$$25 = r^2 \quad (1)$$

$$\sqrt{25} = \sqrt{r^2}$$

$$5 = r$$

Dividing (1) & (2)

$$\frac{4}{3} = \frac{r \sin \phi}{r \cos \phi}$$

$$\frac{4}{3} = \tan \phi$$

$$\tan^{-1} \frac{4}{3} = \phi$$

Then,

$$= r \cos \phi \sin \theta - r \sin \phi \cos \theta$$

$$= r (\sin \theta \cos \phi - \cos \theta \sin \phi)$$

$$= 5 \sin (\theta - \phi)$$

$$= 5 \sin \left(\theta - \tan^{-1} \frac{4}{3} \right) \quad \therefore \phi = \tan^{-1} \frac{4}{3}$$

$$\text{iii) } 1 \sin 0 - 1 \cos 0$$

Using formula

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Let

$$1 = x \cos \phi - 0, \quad 1 = x \sin \phi \quad \text{--- (2)}$$

Adding & Squaring (1) & (2)

$$1^2 + 1^2 = x^2 \cos^2 \phi + x^2 \sin^2 \phi$$

$$2 = x^2 (\cos^2 \phi + \sin^2 \phi)$$

$$2 = x^2 (1)$$

$$\sqrt{2} = x$$

Dividing (1) & (2)

$$\frac{1}{1} = \frac{x \sin \phi}{x \cos \phi}$$

$$1 = \tan \phi$$

$$\tan^{-1} 1 = \phi$$

$$\frac{\pi}{4} = \phi$$

Then,

$$= 8 \cos \phi \sin \theta - 8 \sin \phi \cos \theta$$

$$= 8 (\sin \theta \cos \phi - \cos \theta \sin \phi)$$

$$= \sqrt{2} \sin (\theta - \phi)$$

$$= \sqrt{2} \sin \left(\theta - \frac{\pi}{4} \right)$$

$$\therefore \phi = \frac{\pi}{4}$$

ii) $5 \sin \theta - 4 \cos \theta$

Using Formula,

$$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Let

$$5 = 8 \cos \phi \quad \text{--- (1)}, \quad 4 = 8 \sin \phi \quad \text{--- (2)}$$

Squaring & Adding (1) & (2)

$$5^2 + 4^2 = 8^2 \cos^2 \phi + 8^2 \sin^2 \phi$$

$$25 + 16 = 8^2 (\cos^2 \phi + \sin^2 \phi)$$

$$41 = 8^2 (1)$$

$$\sqrt{41} = \sqrt{8^2}$$

$$\sqrt{41} = 8$$

Dividing ① & ②

$$\frac{4}{5} = \frac{r \sin \phi}{r \cos \phi}$$

$$\frac{4}{5} = \tan \phi$$

$$\tan^{-1} \frac{4}{5} = \phi$$

Then,

$$= r \cos \phi \sin \theta - r \sin \phi \cos \theta$$

$$= r (\sin \theta \cos \phi - \cos \theta \sin \phi)$$

$$= \sqrt{41} \sin(\theta - \phi)$$

$$= \sqrt{41} \sin\left(\theta - \tan^{-1} \frac{4}{5}\right) \because \phi = \tan^{-1} \frac{4}{5}$$

1) $1 \sin \theta + 1 \cos \theta$

Using Formula,

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

Let

$$1 = r \cos \phi \quad \text{--- ①} ; \quad 1 = r \sin \phi \quad \text{--- ②}$$

Squaring & Adding ① & ②

$$1^2 + 1^2 = r^2 \cos^2 \phi + r^2 \sin^2 \phi$$

$$1+1 = r^2 (\cos^2 \phi + \sin^2 \phi)$$

$$2 = r^2 \quad (1)$$

$$\sqrt{2} = r$$

Dividing (1) & (2)

$$\frac{1}{1} = \frac{r \sin \phi}{r \cos \phi}$$

$$1 = \tan \phi$$

$$\tan^{-1} 1 = \phi$$

$$\frac{\pi}{4} = \phi$$

Then,

$$= r \cos \phi \sin \theta + r \sin \phi \cos \theta$$

$$= r (\sin \theta \cos \phi + \cos \theta \sin \phi)$$

$$= \sqrt{2} \sin(\theta + \phi)$$

$$= \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) \quad \therefore \phi = \frac{\pi}{4}$$

$$13) \quad 3\sin\theta - 5\cos\theta$$

Using Formula,

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Let

$$3 = r\cos\phi \quad \text{--- (1)} \quad , \quad 5 = r\sin\phi \quad \text{--- (2)}$$

Squaring & Adding (1) & (2)

$$3^2 + 5^2 = r^2\cos^2\phi + r^2\sin^2\phi$$

$$9 + 25 = r^2(\cos^2\phi + \sin^2\phi)$$

$$34 = r^2 \quad (1)$$

$$\sqrt{34} = r$$

Dividing (1) & (2)

$$\frac{5}{3} = \frac{r\sin\phi}{r\cos\phi}$$

$$\frac{5}{3} = \tan\phi$$

$$\tan^{-1} \frac{5}{3} = \phi$$

$$\begin{aligned} \text{Then,} \quad &= r\cos\phi\sin\theta - r\sin\phi\cos\theta \\ &= r(\sin\theta\cos\phi - \cos\theta\sin\phi) \\ &= \sqrt{34} \sin(\theta - \phi) \\ &= \sqrt{34} \sin\left(\theta - \tan^{-1} \frac{5}{3}\right) \quad \because \phi = \tan^{-1} \frac{5}{3} \end{aligned}$$