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Ex# 10.3

Q.1:-

$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$$ FORMULAS

$$\boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$$

$$= 1 - \sin^2\alpha - \sin^2\alpha$$

$$\boxed{\cos 2\alpha = 1 - 2\sin^2\alpha}$$

$$= \cos^2\alpha - (1 - \cos^2\alpha)$$

$$= \cos^2\alpha - 1 + \cos^2\alpha$$

$$\boxed{\cos 2\alpha = 2\cos^2\alpha - 1}$$

$$\boxed{\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}}$$

i) $\sin\alpha = \frac{12}{13}$

Using Pythagoras theorem

$$(hyp)^2 = (base)^2 + (perp)^2$$

$$(13)^2 = (base)^2 + (12)^2$$

$$169 - 144 = (base)^2$$

$$25 = (base)^2$$

$5 = \text{base}$

$\sin\alpha = \frac{12}{13}$.

$\cos\alpha = \frac{5}{13}$

$\tan\alpha = \frac{12}{5}$.

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{12}{13} \times \frac{5}{13}$

$= \frac{120}{169}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2$

$= \frac{25}{169} - \frac{144}{169}$

$= \frac{-109}{169}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{24}{5}}{1-\frac{144}{25}}$$

$$= \frac{\frac{24}{5}}{\frac{25-144}{25}}$$

$$= \frac{24}{\cancel{5}_1} \times \frac{\cancel{25}^5}{-119}$$

$$= -\frac{120}{119}$$

(iii) $\cos\alpha = \frac{3}{5}$

Using Pythagoras Theorem

$(hyp)^2 = (base)^2 + (perp)^2$

$(5)^2 = (3)^2 + (perp)^2$

$25 - 9 = (perp)^2$

$16 = (perp)^2$

$4 = perp$

$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$\tan\alpha = \frac{4}{3}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{4}{5} \times \frac{3}{5}$

$= \frac{24}{25}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25}$

$= -\frac{7}{25}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\times\frac{4}{3}}{1-\left(\frac{4}{3}\right)^2}$$

$$= \frac{\frac{8}{3}}{\frac{9-16}{9}}$$

$$= \frac{8}{\cancel{3}_{1}} \times \frac{{}^{3}\cancel{9}}{-7}$$

$$= -\frac{24}{7}$$

Q2- L.H.S.

$$= \cot\alpha - \tan\alpha$$

$$= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{\cos^2\alpha - \sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{2\cos 2\alpha}{2\sin\alpha\cos\alpha}$$ $\because \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

Multiply & Divided by 2

$$= \frac{2\cos 2\alpha}{\sin 2\alpha} \qquad \because 2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$= 2\cot 2\alpha$$

$$= R.H.S.$$

Q3. L.H.S.

$$= \frac{\sin 2\alpha}{1+\cos 2\alpha}$$

$$= \frac{2\sin\alpha\cos\alpha}{1+\cos^2\alpha-\sin^2\alpha} \qquad \because \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$= \frac{2\sin\alpha\cos\alpha}{\cos^2\alpha+\cos^2\alpha}$$

$$= \frac{\cancel{2}\sin\alpha\cancel{\cos\alpha}}{\cancel{2}\cos^{\cancel{2}}\alpha}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= R.H.S.$$

Q4:- L.H.S.

$$= \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{1-\cos\alpha}{\sin\alpha} \times \frac{1+\cos\alpha}{1+\cos\alpha}$$

$$= \frac{(1-\cos^2\alpha)}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{\sin^{\cancel{2}}\alpha}{\cancel{\sin\alpha}(1+\cos\alpha)} \qquad \because 1-\cos^2\alpha = \sin^2\alpha$$

$$= \frac{\sin\alpha}{1+\cos\alpha}$$

$$= \frac{\cancel{2}\sin\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{\alpha}{2}} \qquad \boxed{\begin{aligned}&\because \sin\alpha = 2\sin\tfrac{\alpha}{2}\cos\tfrac{\alpha}{2}\\ &1+\cos\alpha = 2\cos^2\tfrac{\alpha}{2}\end{aligned}}$$

$$= \frac{\sin\alpha/2}{\cos\alpha/2}$$

$$= \tan\frac{\alpha}{2}$$

= R.H.S.

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OR

$$L.H.S = \frac{1-\cos\alpha}{\sin\alpha}$$

$$= \frac{\cancel{2}\sin^2\frac{\alpha}{2}}{\cancel{2}\cancel{\sin\frac{\alpha}{2}}\cos\frac{\alpha}{2}}$$

$\because 1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$

$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$$= \frac{\sin\alpha/2}{\cos\alpha/2}$$

$$= \tan\frac{\alpha}{2} = R.H.S.$$

Q5:- L.H.S.

$$= \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha}$$

$$= \frac{\cos\alpha-\sin\alpha}{\cos\alpha+\sin\alpha} \times \frac{\cos\alpha-\sin\alpha}{\cos\alpha-\sin\alpha}$$

$$= \frac{(\cos\alpha-\sin\alpha)^2}{\cos^2\alpha-\sin^2\alpha}$$

$$= \frac{\cos^2\alpha+\sin^2\alpha-2\sin\alpha\cos\alpha}{\cos2\alpha}$$

$\because \cos^2\alpha-\sin^2\alpha = \cos2\alpha$

$$= \frac{1-2\sin\alpha\cos\alpha}{\cos2\alpha}$$

$$= \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \sec 2\alpha - \tan 2\alpha$$

$$= R.H.S.$$

Q6:- L.H.S.

$$= \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}}$$

$$= \sqrt{\frac{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$$

$$= \cancel{\sqrt{\frac{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2} - \cos\frac{\alpha}{2}\right)^2}}}$$

$$= \frac{\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}}{\sin\alpha/2 - \cos\alpha/2}$$

$$= R.H.S$$

Q7:- L.H.S.

$$= \frac{\operatorname{cosec}\theta + 2\operatorname{cosec}2\theta}{\sec\theta}$$

$$= \frac{\frac{1}{\sin\theta} + \frac{2}{\sin 2\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{\sin 2\theta + 2\sin\theta}{\sin\theta \sin 2\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{2\sin\theta\cos\theta + 2\sin\theta}{\sin\theta\, 2\sin\theta\cancel{\cos\theta}}}{\cancel{\frac{1}{\cos\theta}}}$$

$$= \frac{\cancel{2\sin\theta}(\cos\theta + 1)}{\cancel{2\sin\theta}\sin\theta}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\cancel{2}\cos^2\frac{\theta}{2}}{\cancel{2}\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}}$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2}$$

$$= R.H.S.$$

Q8:- L.H.S.

$$= 1 + \tan\alpha \tan 2\alpha$$

$$= 1 + \frac{\sin\alpha}{\cos\alpha}\frac{\sin 2\alpha}{\cos 2\alpha}$$

$$= \frac{\cos 2\alpha\cos\alpha + \sin 2\alpha \sin\alpha}{\cos 2\alpha \cos\alpha}$$

$$= \frac{\cos(2\alpha - \alpha)}{\cos 2\alpha \cos\alpha}$$

$$= \frac{\cancel{\cos\alpha}}{\cos 2\alpha \cancel{\cos\alpha}} = \frac{1}{\cos 2\alpha}$$

$$= \sec 2\alpha$$

$$= R.H.S.$$

Q 9.

L.H.S.

$$= \frac{2\sin\theta \sin 2\theta}{\cos\theta + \cos 3\theta}$$

$$= \frac{2\sin\theta \sin 2\theta}{\cos\theta + 4\cos^3\theta - 3\cos\theta} \qquad \because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$= \frac{2\sin\theta \sin 2\theta}{4\cos^3\theta - 2\cos\theta}$$

$$= \frac{\cancel{2}\sin\theta \sin 2\theta}{\cancel{2}\cos\theta (2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin 2\theta}{\cos 2\theta} \qquad \because 2\cos^2\theta - 1 = \cos 2\theta$$

$$= \tan\theta \tan 2\theta$$

$= R.H.S.$

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Triple Angle Formulae

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\tan 3\alpha = \frac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - (1-\cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1$$

Q10: L.H.S.

$$= \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$$

$$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}}$$

$$= 2$$

$$= R.H.S.$$

Q11.

L.H.S.

$$= \frac{\cos 3\theta}{\cos\theta} + \frac{\sin 3\theta}{\sin\theta}$$

$$= \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta} + \frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}$$

$$= \frac{\cancel{\cos\theta}(4\cos^2\theta - 3)}{\cancel{\cos\theta}} + \frac{\cancel{\sin\theta}(3 - 4\sin^2\theta)}{\cancel{\sin\theta}}$$

$$= 4\cos^2\theta - \cancel{3} + \cancel{3} - 4\sin^2\theta$$

$$= 4\cos^2\theta - 4\sin^2\theta$$

$$= 4(\cos^2\theta - \sin^2\theta)$$

$$= 4\cos 2\theta$$

$$= R.H.S.$$

Q12.

L.H.S.

$$= \frac{\tan\theta/2 + \cot\theta/2}{\cot\theta/2 - \tan\theta/2}$$

$$= \frac{\frac{\sin\theta/2}{\cos\theta/2} + \frac{\cos\theta/2}{\sin\theta/2}}{\frac{\cos\theta/2}{\sin\theta/2} - \frac{\sin\theta/2}{\cos\theta/2}}$$

$$= \frac{\dfrac{\sin^2\theta/2 + \cos^2\theta/2}{\cancel{\cos\theta/2 \sin\theta/2}}}{\dfrac{\cos^2\theta/2 - \sin^2\theta/2}{\cancel{\sin\theta/2 \cos\theta/2}}}$$

$$= \frac{\sin^2\theta/2 + \cos^2\theta/2}{\cos^2\theta/2 - \sin^2\theta/2}$$

$$= \frac{1}{\cos 2\left(\frac{\theta}{2}\right)}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

$= R.H.S.$

Q13:- L.H.S.

$$= \frac{\sin 3\theta}{\cos\theta} + \frac{\cos 3\theta}{\sin\theta}$$

$$= \frac{\sin 3\theta \sin\theta + \cos 3\theta \cos\theta}{\cos\theta \sin\theta}$$

$$= \frac{\cos 3\theta \cos\theta + \sin 3\theta \sin\theta}{\sin\theta \cos\theta}$$ Rearranging

$$= \frac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}$$

$$= \frac{2\cos 2\theta}{2\sin\theta\cos\theta}$$ multiply & Divided by '2'

$$= \frac{2\cos 2\theta}{\sin 2\theta}$$

$$= 2\cot 2\theta$$

$$= R.H.S.$$

Q14: $\sin^4\theta$

$$= (\sin^2\theta)^2$$

$$= \left(\frac{1-\cos 2\theta}{2}\right)^2$$

$$= \frac{(1)^2 + (\cos 2\theta)^2 - 2(1)(\cos 2\theta)}{4}$$

$$= \frac{1}{4}\left[1 + \frac{1+\cos 4\theta}{2} - 2\cos 2\theta\right]$$

$$= \frac{1}{4}\left[\frac{2+1+\cos 4\theta - 4\cos 2\theta}{2}\right]$$

$$= \frac{1}{8}\left[3+\cos 4\theta - 4\cos 2\theta\right]$$

All the functions have single power.

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1 - \sin^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}}$$