

Ex # 10.4

Q11-

$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$

$$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$$

$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$2\sin\alpha\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$$

$$\boxed{2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)}$$

i) $2\sin 30^\circ \cos 0^\circ$

$$= \sin(30^\circ + 0^\circ) + \sin(30^\circ - 0^\circ)$$

$$= \sin 40^\circ + \sin 20^\circ$$

ii) $2\sin 70^\circ \sin 20^\circ$

$$= \sin(50^\circ + 30^\circ) - \sin(50^\circ - 30^\circ)$$

$$= \sin 80^\circ - \sin 20^\circ$$

iii) $\sin 50^\circ \cos 20^\circ$

$$= \frac{1}{2} \times 2\sin 50^\circ \cos 0^\circ$$

$$= \frac{1}{2} [\sin(50^\circ + 0^\circ) + \sin(50^\circ - 0^\circ)]$$

$$= \frac{1}{2} [\sin 60 + \sin 40]$$

$$\begin{aligned} \text{iv)} &= 2 \sin 70 \sin 20 \\ &= -1(-2 \sin 70 \sin 20) \\ &= -[\cos(70+20) - \cos(70-20)] \end{aligned}$$

$$= -\cos 90 + \cos 50$$

$$= \cos 50 - \cos 90$$

OR

$$= 2 \sin 70 \sin 20$$

$$= \cos(70-20) - \cos(70+20)$$

$$= \cos 50 - \cos 90$$

$$\text{v)} \cos(x+y) \sin(x-y)$$

$$= \frac{1}{2} \times 2 \cos(x+y) \sin(x-y)$$

$$= \frac{1}{2} [\sin(x+y+x-y) - \sin(x+y-x-y)]$$

$$= \frac{1}{2} [\sin 2x - \sin 2y]$$

$$vi) \cos(2x+30^\circ) \cos(2x-30^\circ)$$

$$= \frac{1}{2} \times 2 \cos(2x+30^\circ) \cos(2x-30^\circ)$$

$$= \frac{1}{2} [\cos(2x+30^\circ+2x-30^\circ) - \cos(2x+30^\circ-2x+30^\circ)]$$

$$= \frac{1}{2} [\cos 4x - \cos 60^\circ]$$

$$= \frac{1}{2} \left[\cos 4x - \frac{1}{2} \right]$$

$$vii) \sin 12^\circ \sin 46^\circ$$

$$= \frac{1}{2} \times 2 \sin 12^\circ \sin 46^\circ$$

$$= \frac{1}{2} [\cos(12^\circ-46^\circ) - \cos(12^\circ+46^\circ)]$$

$$= \frac{1}{2} [\cos(-34^\circ) - \cos(58^\circ)]$$

$$= \frac{1}{2} [\cos 34^\circ - \cos 58^\circ]$$

$$\text{viii)} \quad \sin(x+45^\circ) - \sin(x-45^\circ)$$

$$= \frac{1}{2} \times 2 \sin(x+45^\circ) \sin(x-45^\circ)$$

$$= \frac{1}{2} \left[\cos(x+45^\circ - x+45^\circ) - \cos(x+45^\circ + x-45^\circ) \right]$$

$$= \frac{1}{2} \left[\cos 90^\circ - \cos 2x \right]$$

$$= \frac{1}{2} \left[0 - \cos 2x \right]$$

$$= -\frac{1}{2} \cos 2x$$

Q2:-

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

$$i) \sin 50^\circ + \sin 30^\circ$$

$$= 2 \sin \left(\frac{50^\circ + 30^\circ}{2} \right) \cos \left(\frac{50^\circ - 30^\circ}{2} \right)$$

$$= 2 \sin \left(\frac{80^\circ}{2} \right) \cos \left(\frac{20^\circ}{2} \right)$$

$$= 2 \sin 40^\circ \cos 10^\circ$$

$$ii) \sin 80^\circ - \sin 40^\circ$$

$$= 2 \cos \left(\frac{80^\circ + 40^\circ}{2} \right) \sin \left(\frac{80^\circ - 40^\circ}{2} \right)$$

$$= 2 \cos \left(\frac{120^\circ}{2} \right) \sin \left(\frac{40^\circ}{2} \right)$$

$$= 2 \cos 60^\circ \sin 20^\circ$$

$$iii) \cos 60^\circ + \cos 30^\circ$$

$$= 2 \cos \left(\frac{60^\circ + 30^\circ}{2} \right) \cos \left(\frac{60^\circ - 30^\circ}{2} \right)$$

$$= 2 \cos \left(\frac{90^\circ}{2} \right) \cos \left(\frac{30^\circ}{2} \right)$$

$$iv) \cos 70^\circ - \cos 0^\circ$$

$$= -2 \sin\left(\frac{70^\circ + 0^\circ}{2}\right) \sin\left(\frac{70^\circ - 0^\circ}{2}\right)$$

$$= -2 \sin\left(\frac{80^\circ}{2}\right) \sin\left(\frac{60^\circ}{2}\right)$$

$$= -2 \sin 40^\circ \sin 30^\circ$$

$$v) \cos 12^\circ + \cos 48^\circ$$

$$= 2 \cos\left(\frac{12^\circ + 48^\circ}{2}\right) \cos\left(\frac{12^\circ - 48^\circ}{2}\right)$$

$$= 2 \cos\left(\frac{60^\circ}{2}\right) \cos\left(\frac{-36^\circ}{2}\right)$$

$$= 2 \cos 30^\circ \cos 18^\circ$$

$$= 2 \times \frac{\sqrt{3}}{2} \cos 18^\circ$$

$$= \sqrt{3} \cos 18^\circ$$

$$v_i) \sin(x+30^\circ) + \sin(x-30^\circ)$$

$$= 2 \sin\left(\frac{x+30^\circ+x-30^\circ}{2}\right) \cos\left(\frac{x+30^\circ-x+30^\circ}{2}\right)$$

$$= 2 \sin\left(\frac{2x}{2}\right) \cos\left(\frac{60^\circ}{2}\right)$$

$$= 2 \sin x \cos 30^\circ$$

$$= 2 \sin x \times \frac{\sqrt{3}}{2}$$

$$= \sqrt{3} \sin x$$

Q3:-

i) L.H.S.

$$\frac{\sin 3x - \sin x}{\cos x - \cos 3x}$$

$$= \frac{2 \cos \left(\frac{3x+x}{2} \right) \sin \left(\frac{3x-x}{2} \right)}{2 \sin \left(\frac{x+3x}{2} \right) \sin \left(\frac{x-3x}{2} \right)}$$

$$= \frac{2 \cos \left(\frac{4x}{2} \right) \sin \left(\frac{2x}{2} \right)}{2 \sin \left(\frac{4x}{2} \right) \sin \left(\frac{-2x}{2} \right)}$$

$$= \frac{\cos 2x \sin x}{\sin 2x (-\sin x)}$$

$$= \frac{\cos 2x \sin x}{\sin 2x \sin x}$$

$$= \cot 2x$$

$$= R.H.S.$$

(c)

L.H.S.

$$= \frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x}$$

$$= \frac{2 \sin \left(\frac{8x+2x}{2} \right) \cos \left(\frac{8x-2x}{2} \right)}{2 \cos \left(\frac{8x+2x}{2} \right) \cos \left(\frac{8x-2x}{2} \right)}$$

$$= \frac{\sin \left(\frac{10x}{2} \right) \cos \left(\frac{6x}{2} \right)}{\cos \left(\frac{10x}{2} \right) \cos \left(\frac{6x}{2} \right)}$$

$$= \frac{\sin 5x \cos 3x}{\cos 5x \cos 3x}$$

$$= \tan 5x$$

$$= R.H.S.$$

$$\text{iii) L.H.S.}$$

$$= \frac{\sin \alpha - \sin \beta}{\sin \alpha + \sin \beta}$$

$$= \frac{2 \cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)}{2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)}$$

$$= \frac{\cos \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)}{\sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)}$$

$$= \frac{\cos \left(\frac{\alpha + \beta}{2} \right)}{\sin \left(\frac{\alpha + \beta}{2} \right)} \times \frac{\sin \left(\frac{\alpha - \beta}{2} \right)}{\cos \left(\frac{\alpha - \beta}{2} \right)}$$

$$= \tan \left(\frac{\alpha - \beta}{2} \right) \cot \left(\frac{\alpha + \beta}{2} \right)$$

$$= \text{R.H.S.}$$

$$\text{Q4) i) L.H.S.}$$

$$= \cos 20^\circ + \cos 100^\circ + \cos 140^\circ$$

$$= 2 \cos \left(\frac{20^\circ + 100^\circ}{2} \right) \cos \left(\frac{20^\circ - 100^\circ}{2} \right) + \cos 140^\circ$$

$$= 2 \cos 60^\circ \cos (-40^\circ) + \cos 140^\circ$$

$$= \frac{1}{2} \cos 40^\circ + \cos 40^\circ$$

$$= \cos 40^\circ + \cos 40^\circ$$

$$= 2 \cos \left(\frac{40^\circ + 40^\circ}{2} \right) \cos \left(\frac{40^\circ - 40^\circ}{2} \right)$$

$$= 2 \cos \left(\frac{180^\circ}{2} \right) \cos \left(\frac{-100^\circ}{2} \right)$$

$$= 2 \cos 90^\circ \cos (-50^\circ)$$

$$= 2 \times 0 \times \cos 50^\circ$$

$$= 0$$

$$= R.H.S.$$

$$(ii) L.H.S.$$

$$= \sin \left(\frac{\pi}{4} - \theta \right) \sin \left(\frac{\pi}{4} + \theta \right)$$

$$= \frac{1}{2} \times -2 \sin \left(\frac{\pi}{4} - \theta \right) \sin \left(\frac{\pi}{4} + \theta \right)$$

$$= \frac{1}{2} \left[\cos \left(\frac{\pi}{4} - \theta + \frac{\pi}{4} + \theta \right) - \cos \left(\frac{\pi}{4} - \theta - \frac{\pi}{4} - \theta \right) \right]$$

$$= \frac{1}{2} \left[\cos\left(\frac{2\pi}{4}\right) - \cos(-2\theta) \right]$$

$$= \frac{1}{2} \left[\cos \frac{\pi}{2} - \cos(2\theta) \right]$$

$$= \frac{1}{2} [0 - \cos 2\theta]$$

$$= \frac{1}{2} \cos 2\theta$$

$$= R.H.S.$$

Q. 11

L.H.S.

$$= \sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta$$

$$\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta$$

$$= 2 \sin\left(\frac{\theta+3\theta}{2}\right) \cos\left(\frac{\theta-3\theta}{2}\right) + 2 \sin\left(\frac{5\theta+7\theta}{2}\right) \cos\left(\frac{5\theta-7\theta}{2}\right)$$

$$= 2 \cos\left(\frac{\theta+3\theta}{2}\right) \cos\left(\frac{\theta-3\theta}{2}\right) + 2 \cos\left(\frac{5\theta+7\theta}{2}\right) \cos\left(\frac{5\theta-7\theta}{2}\right)$$

$$= 2 \sin 2\theta \cos \theta + 2 \sin 6\theta \cos \theta$$

$$= 2 \cos 2\theta \cos \theta + 2 \cos 6\theta \cos \theta$$

$$= \frac{2 \cos 0 (\sin 20 + \sin 60)}{2 \cos 0 (\cos 20 + \cos 60)}$$

$$= \frac{2 \sin \left(\frac{20+60}{2} \right) \cos \left(\frac{20-60}{2} \right)}{2 \cos \left(\frac{20+60}{2} \right) \cos \left(\frac{20-60}{2} \right)}$$

$$= \frac{\sin 40 \cos(-20)}{\cos 40 \cos(-20)}$$

$$= \tan 40$$

$$= \text{R.H.S.}$$

Q.5:-

i) C. A.S.

$$= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$$

$$= \cos 20^\circ \cos 40^\circ \times \frac{1}{2} \times \cos 80^\circ$$

$$= \frac{1}{2 \times 2} \times (2 \cos 20^\circ \cos 40^\circ) \cos 80^\circ$$

Multiply & Divided by 2

$$= \frac{1}{4} [\cos(20^\circ + 40^\circ) + \cos(20^\circ - 40^\circ)] \cos 80^\circ$$

$$= \frac{1}{4} [\cos 60^\circ + \cos(-20^\circ)] \cos 80^\circ$$

$$= \frac{1}{4} \cos 80^\circ \left[\frac{1}{2} + \cos 20^\circ \right]$$

$$= \frac{1}{4} \cos 80^\circ \times \frac{1}{2} + \frac{1}{4} \cos 80^\circ \cos 20^\circ$$

$$= \frac{1}{8} \cos 80^\circ + \frac{1}{4 \times 2} \times 2 \cos 80^\circ \cos 20^\circ$$

$$= \frac{1}{8} \cos 80^\circ + \frac{1}{8} [\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)]$$

$$= \frac{1}{8} \cos 80^\circ + \frac{1}{8} [\cos 100^\circ + \cos 60^\circ]$$

$$= \frac{1}{8} \cos 80^\circ + \frac{1}{8} \left[\cos 100^\circ + \frac{1}{2} \right]$$

$$= \frac{1}{8} \cos 80^\circ + \frac{1}{8} \cos 100^\circ + \frac{1}{16}$$

$$= \frac{1}{8} [\cos 80^\circ + \cos 100^\circ] + \frac{1}{16}$$

$$= \frac{1}{8} \left[2 \cos \left(\frac{80^\circ + 100^\circ}{2} \right) \cos \left(\frac{80^\circ - 100^\circ}{2} \right) \right] + \frac{1}{16}$$

$$= \frac{1}{8} [2 \cos 90^\circ \cos 10^\circ] + \frac{1}{16}$$

$$= \frac{1}{8} [2 \times 0 \times \cos 10^\circ] + \frac{1}{16}$$

$$= 0 + \frac{1}{16}$$

$$= \frac{1}{16}$$

$$= R.H.S.$$

ex) C.H.S.

$$= \sin \frac{\pi}{9} \sin \frac{2\pi}{9} \sin \frac{\pi}{3} \sin \frac{4\pi}{9}$$

$$= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$$

$$= \sin 20^\circ \sin 40^\circ \frac{\sqrt{3}}{2} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{2 \times 2} [2 \sin 20^\circ \sin 40^\circ] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos(20^\circ - 40^\circ) - \cos(20^\circ + 40^\circ)] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos 20^\circ - \cos 60^\circ] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} [\cos 20^\circ - \frac{1}{2}] \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4} \sin 80^\circ \cos 20^\circ - \frac{(\sqrt{3}) \sin 80^\circ}{4 \times 2}$$

$$= \frac{\sqrt{3}}{4 \times 2} [2 \sin 80^\circ \cos 20^\circ] - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} [\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ)] - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} [\sin 100^\circ + \sin 80^\circ] - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \left[\sin 100^\circ + \frac{\sqrt{3}}{2} \right] - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \sin 100^\circ + \frac{(\sqrt{3})^2}{8 \times 2} - \frac{\sqrt{3}}{8} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{8} \sin 100^\circ - \frac{\sqrt{3}}{8} \sin 80^\circ + \frac{3}{16}$$

$$= \frac{\sqrt{3}}{8} [\sin 100^\circ - \sin 80^\circ] + \frac{3}{16}$$

$$= \frac{\sqrt{3}}{8} \left[2 \cos \left(\frac{100^\circ + 80^\circ}{2} \right) \sin \left(\frac{100^\circ - 80^\circ}{2} \right) \right] + \frac{3}{16}$$

$$= \frac{\sqrt{3}}{8} [2 \cos 90^\circ \sin 10^\circ] + \frac{3}{16}$$

$$= \frac{\sqrt{3}}{8} [2 \times 0 \times \sin 10^\circ] + \frac{3}{16}$$

$$= 0 + \frac{3}{16}$$

$$= \frac{3}{16}$$

= R.H.S.

Q. 11.8.

$$= \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$$
$$= \sin 10^\circ \cdot \frac{1}{2} \sin 50^\circ \sin 70^\circ$$

$$= \frac{1}{2 \times 2} (2 \sin 10^\circ \sin 50^\circ) \sin 70^\circ$$

$$= \frac{1}{4} [\cos(10^\circ - 50^\circ) - \cos(10^\circ + 50^\circ)] \sin 70^\circ$$

$$= \frac{1}{4} [\cos(-40^\circ) - \cos 60^\circ] \sin 70^\circ$$

$$= \frac{1}{4} \sin 70^\circ \left[\cos 40^\circ - \frac{1}{2} \right]$$

$$= \frac{1}{4} \sin 70^\circ \cos 40^\circ - \frac{1}{2 \times 4} \sin 70^\circ$$

$$= \frac{1}{4 \times 2} [2 \sin 70^\circ \cos 40^\circ] - \frac{1}{8} \sin 70^\circ$$

$$= \frac{1}{8} [\sin(70^\circ + 40^\circ) + \sin(70^\circ - 40^\circ)] - \frac{1}{8} \sin 70^\circ$$

$$= \frac{1}{8} [\sin 110^\circ + \sin 30^\circ] - \frac{1}{8} \sin 70^\circ$$

$$= \frac{1}{8} \sin 110^\circ + \frac{1}{8} \times \frac{1}{2} - \frac{1}{8} \sin 70^\circ$$

$$= \frac{1}{8} \sin 110^\circ - \frac{1}{8} \sin 70^\circ + \frac{1}{16}$$

$$= \frac{1}{8} [\sin 110^\circ - \sin 70^\circ] + \frac{1}{16}$$

$$= \frac{1}{8} \left[2 \cos \left(\frac{110^\circ + 70^\circ}{2} \right) \sin \left(\frac{110^\circ - 70^\circ}{2} \right) \right] + \frac{1}{16}$$

$$= \frac{1}{8} [2 \cos 90^\circ \sin 20^\circ] + \frac{1}{16}$$

$$= \frac{1}{8} [2 \times 0 \sin 20^\circ] + \frac{1}{16}$$

$$= 0 + \frac{1}{16}$$

$$= \frac{1}{16}$$

- R. H. S.