Ex # 13.1

(1)

i) $\sin^{-1}(-1)$

Let

$$y = \sin^{-1}(1)$$

$$\sin y = 1$$

$$= \frac{\pi}{2}$$

ii) $\sin^{-1}(-1)$

Let

$$y = \sin^{-1}(-1)$$

$$\sin y = -1$$

$$= -\frac{\pi}{2}$$

iii) $\cos^{-1}\frac{\sqrt{3}}{2}$

Let

$$y = \cos^{-1}\frac{\sqrt{3}}{2}$$

$$\cos y = \frac{\sqrt{3}}{2}$$

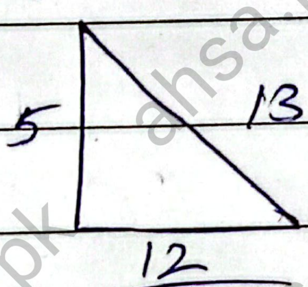
$$= \frac{\pi}{6}$$

②

i) $\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$

Let $\tan^{-1}\frac{5}{12} = \alpha$

$\frac{5}{12} = \tan\alpha$



Using Pythagorus Theorem.

$(hyp)^2 = (base)^2 + (perp)^2$

$(hyp)^2 = (12)^2 + (5)^2$

$(hyp)^2 = 144 + 25$

$(hyp)^2 = 169$

$(hyp)^2 = 169$

$hyp = 13$

$\sin\alpha = \frac{P}{H}$

$\sin\alpha = \frac{5}{13}$

$$\alpha = \sin^{-1}\frac{5}{13}$$

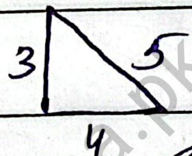
By putting the value of 'α'

$$\tan^{-1}\frac{5}{12} = \sin^{-1}\frac{5}{13}$$

(iii) $$2\cos^{-1}\frac{4}{5} = \sin^{-1}\frac{24}{25}$$

Let $\cos^{-1}\frac{4}{5} = \alpha$

$$\frac{4}{5} = \cos\alpha$$



Using Pythagorus Theorem

$$(hyp)^2 = (base)^2 + (perp)^2$$

$$(5)^2 = (4)^2 + (perp)^2$$

$$25 - 16 = (perp)^2$$

$$9 = (perp)^2$$

$$3 = perp$$

$$\sin\alpha = \frac{P}{H}$$

$$\sin\alpha = \frac{3}{5}$$

Then,

$$2\alpha = \sin^{-1}\frac{24}{25}$$

$$\sin 2\alpha = \frac{24}{25}$$

$$2\sin\alpha\cos\alpha = \frac{24}{25}$$

$$2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

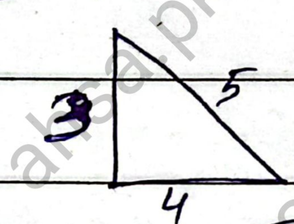
$$\frac{24}{25} = \frac{24}{25}$$

Hence Proved

iii) $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$

Let $\cos^{-1}\frac{4}{5} = \alpha$

$$\frac{4}{5} = \cos\alpha$$



Using Pythagorus Theorem.

$(hyp)^2 = (base)^2 + (perp)^2$

$(5)^2 = (4)^2 + (perp)^2$

$25 - 16 = (perp)^2$

$9 = (perp)^2$

$3 = perp$

$\tan\alpha = \frac{P}{B}$ and $\cot\alpha = \frac{B}{P}$

$\cot\alpha = \frac{4}{3}$

$\alpha = \cot^{-1}\frac{4}{3}$

ii) $\sec\left(\cos^{-1}\frac{1}{2}\right)$

Let $y = \cos^{-1}\frac{1}{2}$

$\cos y = \frac{1}{2}$

$y = \frac{\pi}{3}$ $\quad \because \cos\frac{\pi}{3} = \frac{1}{2}$

Then

$= \sec\left(\cos^{-1}\frac{1}{2}\right)$

$= \sec y$

$= \frac{1}{\cos y} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

iv) $\operatorname{cosec}(\tan^{-1}(-1))$

Let

$y = \tan^{-1}(-1)$

$\tan y = -1$

$y = -\frac{\pi}{4}$ $\because \tan\left(-\frac{\pi}{4}\right) = -1$

Then

$= \operatorname{cosec} y$

$= \frac{1}{\sin y}$

$= \frac{1}{\sin\left(-\frac{\pi}{4}\right)}$

$= \frac{1}{-\frac{1}{\sqrt{2}}}$

$= -\sqrt{2}$

v) $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

Let

$$y = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\sin y = -\frac{1}{2}$$

$$y = -\frac{\pi}{6} \qquad \because \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$$

Then

$$\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \sec y$$

$$= \sec\left(-\frac{\pi}{6}\right)$$

$$= \sec\frac{\pi}{6}$$

$$= \frac{1}{\cos\frac{\pi}{6}}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}}$$

$$= \frac{2}{\sqrt{3}}$$

vi) $\tan(\tan^{-1}(-1))$

let

$y = \tan^{-1}(-1)$

$\tan y = -1$

$y = -\frac{\pi}{4}$ $\because \tan\left(-\frac{\pi}{4}\right) = -1$

Then

$= \tan y$

$= \tan\left(-\frac{\pi}{4}\right)$

$= -1$

vii) $\sin\left(\sin^{-1}\frac{1}{2}\right)$

let

$y = \sin^{-1}\frac{1}{2}$

$\sin y = \frac{1}{2}$

$y = \frac{\pi}{6}$ $\because \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Then

$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin y$

$= \sin\frac{\pi}{6}$

$= \frac{1}{2}$

viii) $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

Let

$y = \sin^{-1}\left(-\frac{1}{2}\right)$

$\sin y = -\frac{1}{2}$

$y = -\frac{\pi}{6}$ $\because \sin\left(\frac{-\pi}{6}\right) = -\frac{1}{2}$

Then

$\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan y$

$$= \tan\left(-\frac{\pi}{6}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

ix) $\sin(\tan^{-1}(-1))$

let

$y = \tan^{-1}(-1)$

$\tan y = -1$

$y = -\frac{\pi}{4}$ $\quad \because \tan\left(-\frac{\pi}{4}\right) = -1$

Then

$\sin(\tan^{-1}(-1)) = \sin y$

$= \sin\left(-\frac{\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}}$