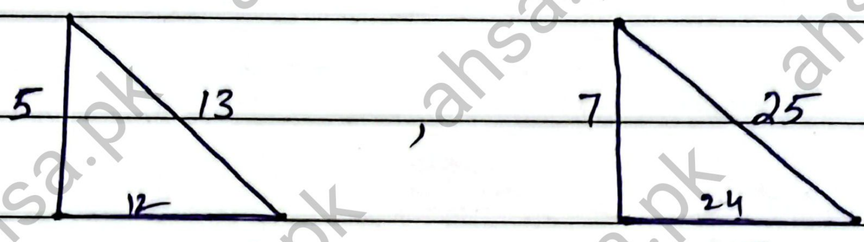
Ex # 13.2

(1) $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$

Let

$$\alpha = \sin^{-1}\frac{5}{13} \quad , \quad \beta = \sin^{-1}\frac{7}{25}$$

$$\sin\alpha = \frac{5}{13} \quad , \quad \sin\beta = \frac{7}{25}$$



Using Pythagorus Theorem

$(hyp)^2 = (base)^2 + (perp)^2 \quad , \quad (hyp)^2 = (base)^2 + (perp)^2$

$13^2 = (base)^2 + 5^2 \quad , \quad 25^2 = (base)^2 + 7^2$

$169 - 25 = (base)^2 \quad , \quad 625 - 49 = (base)^2$

$144 = (base)^2 \quad , \quad 576 = (base)^2$

$12 = base \quad , \quad 24 = base$

$\cos\alpha = \frac{B}{H} = \frac{12}{13} \quad , \quad \cos\beta = \frac{B}{H} = \frac{24}{25}$

Then,

$$\alpha + \beta = \cos^{-1}\frac{253}{325}$$

$$\cos(\alpha+\beta) = \frac{253}{325}$$

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{253}{325}$$

$$\frac{12}{13} \times \frac{24}{25} - \frac{5}{13} \times \frac{7}{25} = \frac{253}{325}$$

$$\frac{288}{325} - \frac{35}{325} = \frac{253}{325}$$

$$\frac{253}{325} = \frac{253}{325}$$

Hence Proved

(7) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} = \tan^{-1}\frac{9}{19}$

L.H.S.

$$= \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5}$$

$$= \tan^{-1}\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{1}{4}\times\frac{1}{5}}$$

$$= \tan^{-1} \frac{\frac{5+4}{20}}{\frac{20-1}{20}}$$

$$= \tan^{-1} \frac{9}{19}$$

$$= R.H.S.$$

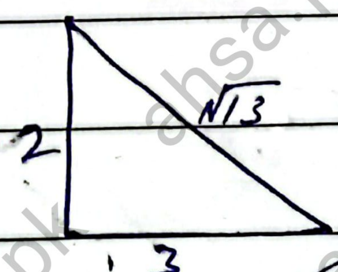
Hence Proved

(3) $2\tan^{-1}\frac{2}{3} = \sin^{-1}\frac{12}{13}$

Let

$$\alpha = \tan^{-1}\frac{2}{3}$$

$$\tan\alpha = \frac{2}{3}$$



Using Pythagoras Theorem.

$$(hyp)^2 = (base)^2 + (perp)^2$$

$$(hyp)^2 = 3^2 + 2^2$$

$$(hyp)^2 = 13$$

$hyp = \sqrt{13}$

then,

$\sin\alpha = \frac{P}{H} = \frac{2}{\sqrt{13}}$, $\cos\alpha = \frac{B}{H} = \frac{3}{\sqrt{13}}$

$$2\alpha = \sin^{-1}\frac{12}{13}$$

$$\sin 2\alpha = \frac{12}{13}$$

$$2\sin\alpha\cos\alpha = \frac{12}{13}$$

$$2 \times \frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}} = \frac{12}{13}$$

$$\frac{12}{(\sqrt{13})^2} = \frac{12}{13}$$

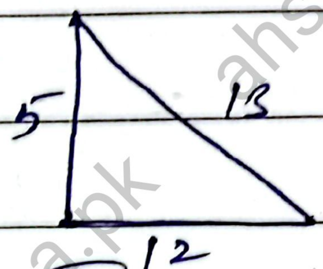
$$\frac{12}{13} = \frac{12}{13}$$

Hence Proved

(4) $\tan^{-1}\frac{120}{119} = 2\cos^{-1}\frac{12}{13}$

Let $\alpha = \cos^{-1}\frac{12}{13}$

$\cos\alpha = \frac{12}{13}$



Using Pythagorus Theorem.

$(hyp)^2 = (base)^2 + (perp)^2$

$13^2 = 12^2 + (perp)^2$

$169 - 144 = (perp)^2$

$25 = (perp)^2$

$\boxed{5 = perp}$

$\tan\alpha = \frac{P}{B} = \frac{5}{12}$

Then,

$\tan^{-1}\frac{120}{119} = 2\alpha$

$\frac{120}{119} = \tan 2\alpha$

$$\frac{120}{119} = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$= \frac{2\left(\frac{5}{12}\right)}{1-\left(\frac{5}{12}\right)^2}$$

$$= \frac{\frac{10}{12}}{1-\frac{25}{144}}$$

$$= \frac{\frac{10}{12}}{\frac{144-25}{144}}$$

$$= \frac{\frac{10}{12}}{\frac{119}{144}}$$

$$= \frac{10}{\cancel{12}_{1}} \times \frac{\cancel{144}^{12}}{119}$$

$$= \frac{120}{119}$$

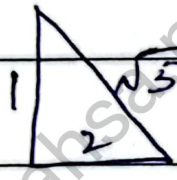
Hence Proved

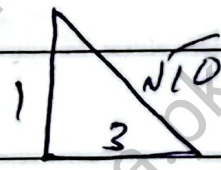
(5) $\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \frac{\pi}{4}$

$$\sin^{-1}\frac{1}{\sqrt{5}} + \cot^{-1}3 = \sin^{-1}\frac{1}{\sqrt{2}}$$

Let $\alpha = \sin^{-1}\frac{1}{\sqrt{5}}$, $\beta = \cot^{-1}3$

$\sin\alpha = \frac{1}{\sqrt{5}}$, $\cot\beta = 3 \Rightarrow \tan\beta = \frac{1}{3}$





Using Pythagorous Theorem

$H^2 = B^2 + P^2$, $H^2 = B^2 + P^2$

$(\sqrt{5})^2 = B^2 + 1^2$, $H^2 = 3^2 + 1^2$

$5 - 1 = B^2$, $H^2 = 9 + 1$

$4 = B^2$, $H^2 = 10$

$2 = B$, $H = \sqrt{10}$

$\cos\alpha = \frac{2}{\sqrt{5}}$, $\sin\beta = \frac{1}{\sqrt{10}}$, $\cos\beta = \frac{3}{\sqrt{10}}$

Then

$$\alpha + \beta = \sin^{-1}\frac{1}{\sqrt{2}}$$

$$\sin(\alpha+\beta)=\frac{1}{\sqrt{2}}$$

$$\sin\alpha\cos\beta+\cos\alpha\sin\beta=\frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}=\frac{1}{\sqrt{2}}$$

$$\frac{3}{\sqrt{50}}+\frac{2}{\sqrt{50}}=\frac{1}{\sqrt{2}}$$

$$\frac{5}{\sqrt{50}}=\frac{1}{\sqrt{2}}$$

$$\frac{\cancel{5}}{\cancel{5}\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

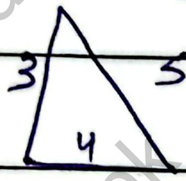
Hence Proved

(6) $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$

Let

$$\alpha = \sin^{-1}\frac{3}{5} \quad , \quad \beta = \sin^{-1}\frac{8}{17}$$

$$\sin\alpha = \frac{3}{5} \quad , \quad \sin\beta = \frac{8}{17}$$



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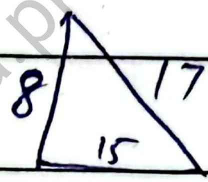


Using Pythagorus Theorem

$(hyp)^2 = (base)^2 + (perp)^2 \quad , \quad (hyp)^2 = (base)^2 + (perp)^2$

$5^2 = (base)^2 + 3^2 \quad , \quad 17^2 = (base)^2 + (8)^2$

$25 - 9 = (base)^2 \quad , \quad 289 - 64 = (base)^2$

$16 = (base)^2 \quad , \quad 225 = (base)^2$

$4 = base \quad , \quad 15 = base$

$$\cos\alpha = \frac{B}{H} = \frac{4}{5} \quad , \quad \cos\beta = \frac{B}{H} = \frac{15}{17}$$

Then,

$$\alpha + \beta = \sin^{-1}\frac{77}{85}$$

$$\sin(\alpha+\beta) = \frac{77}{85}$$

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{77}{85}$$

$$\frac{3}{5} \times \frac{15}{17} + \frac{4}{5} \times \frac{8}{17} = \frac{77}{85}$$

$$\frac{45}{85} + \frac{32}{85} = \frac{77}{85}$$

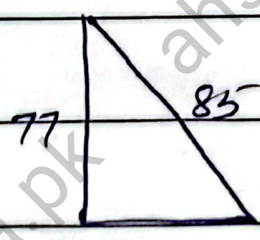
$$\frac{77}{85} = \frac{77}{85}$$

Hence Proved.

(7) $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$

Let $\alpha = \sin^{-1}\frac{77}{85}$, $\beta = \sin^{-1}\frac{3}{5}$

$\sin\alpha = \frac{77}{85}$, $\sin\beta = \frac{3}{5}$



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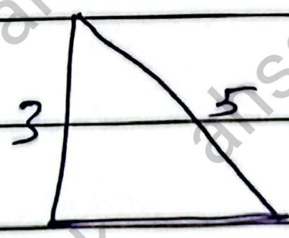


$H^2 = B^2 + P^2$, $H^2 = B^2 + P^2$

$85^2 = B^2 + 77^2$, $5^2 = B^2 + 3^2$

$7225 - 5929 = B^2$, $25 - 9 = B^2$

$1296 = B^2$, $16 = B^2$

$36 = B$, $4 = B$

$\cos\alpha = \frac{B}{H} = \frac{36}{85}$, $\cos\beta = \frac{B}{H} = \frac{4}{5}$

Then,

$$\alpha - \beta = \cos^{-1}\frac{15}{17}$$

$$\cos(\alpha - \beta) = \frac{15}{17}$$

$$\cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{15}{17}$$

$$\frac{36}{85} \times \frac{4}{5} + \frac{77}{85} \times \frac{3}{5} = \frac{15}{17}$$

$$\frac{144}{425} + \frac{231}{425} = \frac{15}{17}$$

$$\frac{375}{425} = \frac{15}{17}$$

$$\frac{15}{17} = \frac{15}{17}$$

Hence Proved

(8) $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{\frac{1}{5}+\frac{1}{5}}{1-\frac{1}{5}\times\frac{1}{5}} = \sin^{-1}\frac{3}{5}$$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{\frac{1+1}{5}}{\frac{25-1}{25}} = \sin^{-1}\frac{3}{5}$$

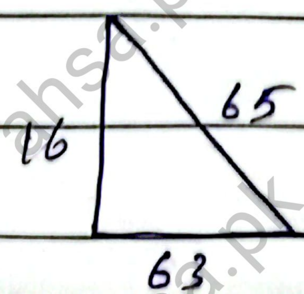
$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{2}{\frac{24}{5}} = \sin^{-1}\frac{3}{5}$$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{10}{24} = \sin^{-1}\frac{3}{5}$$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12} = \sin^{-1}\frac{3}{5}$$

Let $\alpha = \cos^{-1}\frac{63}{65}$, $\beta = \tan^{-1}\frac{5}{12}$

$\cos\alpha = \frac{63}{65}$, $\tan\beta = \frac{5}{12}$



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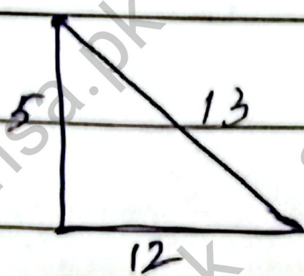

Using Pythagoras Theorem

$H^2 = B^2 + P^2$, $H^2 = B^2 + P^2$

$65^2 = 63^2 + P^2$, $H^2 = 12^2 + 5^2$

$4225 - 3969 = P^2$, $H^2 = 144 + 25$

$256 = P^2$, $H^2 = 169$

$16 = P$, $H = 13$

$\sin\alpha = \frac{P}{H} = \frac{16}{65}$, $\sin\beta = \frac{P}{H} = \frac{5}{13}$

$\cos\beta = \frac{B}{H} = \frac{12}{13}$

Then,

$\alpha + \beta = \sin^{-1}\frac{3}{5}$

$\sin(\alpha+\beta) = \frac{3}{5}$

$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{3}{5}$

$\frac{16}{65} \times \frac{12}{13} + \frac{63}{65} \times \frac{5}{13} = \frac{3}{5}$

$\frac{192}{845} + \frac{315}{845} = \frac{3}{5}$

$$\frac{507}{845} = \frac{3}{5}$$

$$\frac{3}{5} = \frac{3}{5}$$

Hence Proved

(9) $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$

L.H.S.

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\times\frac{3}{5}} - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\frac{\frac{15+12}{\cancel{20}}}{\frac{20-9}{\cancel{20}}} - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\frac{27}{11} - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\frac{\frac{27}{11}-\frac{8}{19}}{1+\frac{27}{11}\times\frac{8}{19}}$$

$$= \tan^{-1} \frac{\frac{513-88}{209}}{\frac{209+216}{209}}$$

$$= \tan^{-1} \frac{\cancel{425}}{\cancel{425}} = \tan^{-1}(1) = \frac{\pi}{4}$$

$$= R.H.S.$$

(10) $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$

L.H.S.

$$= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{4}{5}\right)^2}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{16}{25}}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{\frac{169-25}{169}} + \frac{5}{13}\sqrt{\frac{25-16}{25}}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\sqrt{\frac{144}{169}} + \frac{5}{13}\sqrt{\frac{9}{25}}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{4}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{3}{5}\right) + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{48}{65} + \frac{15}{65}\right) + \sin^{-1}\frac{16}{65}$$

$$\sin^{-1}\alpha + \sin^{-1}\beta = \sin^{-1}\left(\alpha\sqrt{1-\beta^2} + \beta\sqrt{1-\alpha^2}\right)$$

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$$= \sin^{-1}\frac{63}{65} + \sin^{-1}\frac{16}{65}$$

$$= \sin^{-1}\left(\frac{63}{65}\sqrt{1-\left(\frac{16}{65}\right)^2} + \frac{16}{65}\sqrt{1-\left(\frac{63}{65}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{63}{65}\sqrt{1-\frac{256}{4225}} + \frac{16}{65}\sqrt{1-\frac{3969}{4225}}\right)$$

$$= \sin^{-1}\left(\frac{63}{65}\sqrt{\frac{4225-256}{4225}} + \frac{16}{65}\sqrt{\frac{4225-3969}{4225}}\right)$$

$$= \sin^{-1}\left(\frac{63}{65}\sqrt{\frac{3969}{4225}} + \frac{16}{65}\sqrt{\frac{256}{4225}}\right)$$

$$= \sin^{-1}\left(\frac{63}{65}\times\frac{63}{65} + \frac{16}{65}\times\frac{16}{65}\right)$$

$$= \sin^{-1}\left(\frac{3969}{4225} + \frac{16\times16}{4225}\right)$$

$$= \sin^{-1}\left(\frac{3969+256}{4225}\right)$$

$$= \sin^{-1}\left(\frac{4225}{4225}\right)$$

$$= \sin^{-1}(1)$$

$$= \frac{\pi}{2} = R.H.S$$

(11) $\tan^{-1}\frac{1}{11}+\tan^{-1}\frac{5}{6}=\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$

$$\tan^{-1}\frac{\frac{1}{11}+\frac{5}{6}}{1-\frac{1}{11}\times\frac{5}{6}}=\tan^{-1}\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}$$

$$\tan^{-1}\frac{\frac{6+55}{66}}{\frac{66-5}{66}}=\tan^{-1}\frac{\frac{2+3}{6}}{\frac{6-1}{6}}$$

$$\tan^{-1}\frac{61}{61}=\tan^{-1}\frac{5}{5}$$

$$\tan^{-1}(1)=\tan^{-1}(1)$$

$$\frac{\pi}{4}=\frac{\pi}{4}$$

Hence Proved

(12) $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\times\frac{1}{3}}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{\frac{1+1}{3}}{\frac{9-1}{9_3}}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{2}{\frac{8}{3}}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{6}{8}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{\frac{21+4}{\cancel{28}}}{\frac{28-3}{\cancel{28}}}=\frac{\pi}{4}$$

$$\tan^{-1}\frac{25}{25}=\frac{\pi}{4}$$

$$\tan^{-1}(1)=\frac{\pi}{4}$$

$$\frac{\pi}{4}=\frac{\pi}{4}$$

Hence Proved

(13) $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

Let

$\alpha = \sin^{-1}x$

$\sin\alpha = x$

Using, $\cos^2\alpha = 1 - \sin^2\alpha$

$\sqrt{\cos^2\alpha} = \sqrt{1-\sin^2\alpha}$

$\cos\alpha = \sqrt{1-x^2}$

By putting the value of 'α'

$\cos(\sin^{-1}x) = \sqrt{1-x^2}$

Hence Proved

(14) $\sin(2\cos^{-1}x) = 2x\sqrt{1-x^2}$

Let

$\alpha = \cos^{-1}x$

$\cos\alpha = x$

Using,

$\sin^2\alpha = 1 - \cos^2\alpha$

$\sqrt{\sin^2\alpha} = \sqrt{1-\cos^2\alpha}$

$\sin\alpha = \sqrt{1-x^2}$

Then,

$$\sin 2\alpha = 2x\sqrt{1-x^2}$$

$$2\sin\alpha\cos\alpha = 2x\sqrt{1-x^2}$$

By putting the values

$$2\sqrt{1-x^2}\,x = 2x\sqrt{1-x^2}$$

$$2x\sqrt{1-x^2} = 2x\sqrt{1-x^2}$$

Hence Proved

(15) $\cos(2\sin^{-1}x) = 1-2x^2$

Let

$\alpha = \sin^{-1}x$

$\sin\alpha = x$

~~Using,~~ No Need

$$\cancel{\cos^2\alpha = 1-\sin^2\alpha}$$ in this

$$\cancel{\sqrt{\cos^2\alpha} = \sqrt{1-\sin^2\alpha}}$$ method

$$\cancel{\cos\alpha = \sqrt{1-x^2}}$$

Then

$$\cos 2\alpha = 1-2x^2$$

$$1-2\sin^2\alpha = 1-2x^2$$

By putting the values

$$1-2x^2 = 1-2x^2$$

Hence Proved

(16) $\tan^{-1}(-x) = -\tan^{-1}x$

$\tan^{-1}(-x) + \tan^{-1}x = 0$

$\tan^{-1}\dfrac{-x+x}{1-(-x)(x)} = 0$

$\tan^{-1}\dfrac{0}{1+x^2} = 0$

$\tan^{-1}0 = 0$

$0 = 0$

Hence Proved

(17) $\sin^{-1}(-x) = -\sin^{-1}(x)$

$\sin^{-1}(-x) + \sin^{-1}x = 0$

$\Rightarrow \sin^{-1}\left(-x\sqrt{1-x^2} + x\sqrt{1-(-x)^2}\right) = 0$

$\Rightarrow \sin^{-1}\left(\cancel{-x\sqrt{1-x^2}} + \cancel{x\sqrt{1-x^2}}\right) = 0$

$\Rightarrow \sin^{-1}(0) = 0$

$\Rightarrow 0 = 0$

Hence Proved

(18) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cos^{-1}(-x) + \cos^{-1}x = \pi$

$\cos^{-1}\left((-x)(x) - \sqrt{(1-(-x)^2)(1-x^2)}\right) = \pi$

$\cos^{-1}\left(-x^2 - \sqrt{(1-x^2)(1-x^2)}\right) = \pi$

$\cos^{-1}\left(-x^2 - \sqrt{(1-x^2)^2}\right) = \pi$

$\cos^{-1}\left(\cancel{-x^2} - 1 + \cancel{x^2}\right) = \pi$

$\cos^{-1}(-1) = \pi$

$\pi = \pi$

Hence Proved.

(19) $\tan(\sin^{-1}x) = \dfrac{x}{\sqrt{1-x^2}}$

Let

$\alpha = \sin^{-1}x$

$\sin\alpha = x$

Using,

$\cos^2\alpha = 1 - \sin^2\alpha$

$\sqrt{\cos^2\alpha} = \sqrt{1-\sin^2\alpha}$

$\cos\alpha = \sqrt{1-x^2}$

Then,

$$\tan\alpha = \frac{x}{\sqrt{1-x^2}}$$

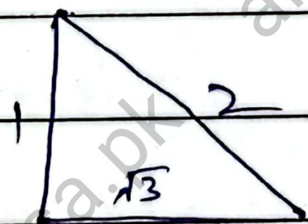
$$\frac{\sin\alpha}{\cos\alpha} = \frac{x}{\sqrt{1-x^2}}$$

$$\frac{x}{\sqrt{1-x^2}} = \frac{x}{\sqrt{1-x^2}}$$

Hence Proved

(20) $x = \sin^{-1}\frac{1}{2}$

$\sin x = \frac{1}{2}$



Using Pythagorus Theorem

$H^2 = B^2 + P^2$

$2^2 = B^2 + 1^2$

$4 - 1 = B^2$

$3 = B^2$

$\sqrt{3} = B$

$\sin x = \frac{P}{H} = \frac{1}{2}$, $\operatorname{cosec} x = \frac{H}{P} = \frac{2}{1}$

$\cos x = \frac{B}{H} = \frac{\sqrt{3}}{2}$, $\sec x = \frac{H}{B} = \frac{2}{\sqrt{3}}$

$\tan x = \frac{P}{B} = \frac{1}{\sqrt{3}}$, $\cot x = \frac{B}{P} = \frac{\sqrt{3}}{1}$