Ex # 9.4

Q1:- L.H.S.

$$= \tan\theta + \cot\theta$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \qquad \because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \qquad \because \sin^2\theta + \cos^2\theta = 1$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$= \operatorname{cosec}\theta \sec\theta \qquad \because \frac{1}{\sin\theta} = \csc\theta, \frac{1}{\cos\theta} = \sec\theta$$

= R.H.S

Q2:- L.H.S.

$$= \sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta$$

$$= \frac{1}{\cancel{\cos\theta}} \times \frac{1}{\cancel{\sin\theta}} \times \cancel{\sin\theta}\cancel{\cos\theta} \qquad \because \sec\theta = \frac{1}{\cos\theta}, \csc = \frac{1}{\sin\theta}$$

$$= 1$$

= R.H.S.

Q3:- L.H.S.

$= \cos\theta + \tan\theta \sin\theta$

$= \cos\theta + \frac{\sin\theta}{\cos\theta} \sin\theta$ $\quad \because \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$ $\quad \because \sin^2\theta + \cos^2\theta = 1$

$= \sec\theta$ $\quad \because \frac{1}{\cos\theta} = \sec\theta$

$=$ R.H.S.

Q4:- L.H.S. $= \mathrm{cosec}\,\theta + \tan\theta \sec\theta$

$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$ $\quad \because \csc\theta = \frac{1}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}, \sec\theta = \frac{1}{\cos\theta}$

$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta}$

$= \frac{1}{\sin\theta\cos^2\theta}$ $\quad \because \cos^2\theta + \sin^2\theta = 1$

$= \operatorname{cosec}\theta \sec^2\theta$ $\qquad \because \frac{1}{\sin\theta} = \operatorname{cosec}\theta, \frac{1}{\cos\theta} = \sec\theta$

$= R.H.S.$

Q5:- L.H.S.

$= \sec^2\theta - \operatorname{cosec}^2\theta$

$= 1+\tan^2\theta - (1+\cot^2\theta)$ $\qquad \because \sec^2\theta = 1+\tan^2\theta$

$\operatorname{cosec}^2\theta = 1+\cot^2\theta$

$= \cancel{1}+\tan^2\theta - \cancel{1} - \cot^2\theta$

$= \tan^2\theta - \cot^2\theta$

$= R.H.S.$

Q6:- L.H.S.

$\cot^2\theta - \cos^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$ $\qquad \because \cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \frac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}$

$= \frac{\cos^2\theta\cos^2\theta}{\sin^2\theta}$ $\qquad \because 1-\sin^2\theta = \cos^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta} \times \cos^2\theta$

$= \cot^2\theta \cos^2\theta$ $\quad \because \frac{\cos\theta}{\sin\theta} = \cot\theta$

$=$ R.H.S.

Q7- L.H.S.

$= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$ $\quad \because (a+b)(a-b) = a^2 - b^2$

$= 1 + \cancel{\tan^2\theta} - \cancel{\tan^2\theta}$ $\quad \because \sec^2\theta = 1 + \tan^2\theta$

$= 1$

$=$ R.H.S.

L.H.S.

Q8:- $= 2\cos^2\theta - 1$

NO WAY

$\boxed{\cancel{= 2\cos^2\theta - (\sin^2\theta + \cos^2\theta)}}$

$= 2(1 - \sin^2\theta) - 1$ $\quad \because \cos^2\theta = 1 - \sin^2\theta$

$= 2 - 2\sin^2\theta - 1$

$= 2 - 1 - 2\sin^2\theta$

$= 1 - 2\sin^2\theta$

$=$ R.H.S.

Q9.- R.H.S.

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cancel{\cos^2\theta}}}{\frac{\cos^2\theta+\sin^2\theta}{\cancel{\cos^2\theta}}}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{1} \qquad \because \cos^2\theta+\sin^2\theta=1$$

$$= \cos^2\theta - \sin^2\theta$$

$= L.H.S.$

Q10.- R.H.S.

$$= \frac{\cot\theta-1}{\cot\theta+1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}+1} \qquad \because \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\frac{\cos\theta - \sin\theta}{\sin\theta}}{\frac{\cos\theta + \sin\theta}{\sin\theta}}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$= L.H.S.$

Q11- L.H.S.

$$= \frac{\sin\theta}{1+\cos\theta} + \cot\theta$$

$$= \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} \quad \because \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\cancel{1+\cos\theta}}{\cancel{(1+\cos\theta)}\sin\theta} \qquad \because \sin^2\theta+\cos^2\theta = 1$$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta \qquad \because \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$= R.H.S.$

Q12:-

L.H.S:

$$= \frac{\cot^2\theta - 1}{1+\cot^2\theta}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\frac{\cos^2\theta - \sin^2\theta}{\cancel{\sin^2\theta}}}{\frac{\sin^2\theta+\cos^2\theta}{\cancel{\sin^2\theta}}}$$

$$= \frac{\cos^2\theta - (1-\cos^2\theta)}{1} \qquad \because \sin^2\theta = 1-\cos^2\theta,$$

$$\sin^2\theta+\cos^2\theta = 1$$

$= \cos^2\theta - 1 + \cos^2\theta$

$= 2\cos^2\theta - 1$

$= R.H.S.$

Q13:-

R.H.S.

$= (\operatorname{cosec}\theta + \cot\theta)^2$

$= \left(\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\right)^2$ $\because \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}, \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$= \left(\dfrac{1+\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{(1+\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}$ $\because \sin^2\theta = 1 - \cos^2\theta$

$= \dfrac{(1+\cos\theta)^{\cancel{2}}}{\cancel{(1+\cos\theta)}(1-\cos\theta)}$ $\because a^2 - b^2 = (a+b)(a-b)$

$(1)^2 - \cos^2\theta = (1+\cos\theta)(1-\cos\theta)$

$= \dfrac{1+\cos\theta}{1-\cos\theta}$

$= L.H.S.$

Q14- L.H.S.

$= (\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$ $\because \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$ $\because \cos^2\theta = 1-\sin^2\theta$

$= \frac{(1-\sin\theta)^{\cancel{2}}}{(1+\sin\theta)\cancel{(1-\sin\theta)}}$

$= \frac{1-\sin\theta}{1+\sin\theta}$

$=$ R.H.S

Q15:- L.H.S.

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{\frac{2\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} \quad \because \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{2\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cos^{\cancel{2}}\theta}} \quad \because \cos^2\theta+\sin^2\theta=1$$

$$= \frac{2\sin\theta}{\frac{1}{\cos\theta}}$$

$$= 2\sin\theta \times \frac{\cos\theta}{1}$$

$$= 2\sin\theta\cos\theta$$

$$= R.H.S.$$

Q16:- $\frac{1-\sin\theta}{\cos\theta}\left(\frac{\cos\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{\cos\theta}\right)=\frac{\cos\theta}{1+\sin\theta}$

L.H.S. $=\frac{1-\sin\theta}{\cos\theta}$

$=\frac{1-\sin\theta}{\cos\theta}\times\frac{1+\sin\theta}{1+\sin\theta}$

$=\frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$ $\because 1-\sin^2\theta=\cos^2\theta$

$=\frac{\cos\theta}{1+\sin\theta}$

$=$ R.H.S

Q17:- $(\tan\theta+\cot\theta)^2=\sec^2\theta\,\mathrm{cosec}^2\theta$

L.H.S.

$=(\tan\theta+\cot\theta)^2$

$=\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$ $\because \tan\theta=\frac{\sin\theta}{\cos\theta}$,

$\cot\theta=\frac{\cos\theta}{\sin\theta}$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta \sin\theta}\right)^2 \quad \because \sin^2\theta + \cos^2\theta = 1$$

$$= \sec^2\theta \operatorname{cosec}^2\theta \quad \because \frac{1}{\cos\theta} = \sec\theta, \frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$= R.H.S.$

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Q 18:- L.H.S.

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} \times \frac{\tan\theta + \sec\theta}{\tan\theta + \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta + \sec\theta)}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{\tan^2\theta + \cancel{\tan\theta\sec\theta} - \cancel{\tan\theta\sec\theta} - \sec^2\theta + \tan\theta + \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{\tan^2\theta - \sec^2\theta + \tan\theta + \sec\theta}$$

$$\boxed{\begin{aligned} &\tan^2\theta - (\tan^2\theta + 1) \\ &\cancel{\tan^2\theta} - \cancel{\tan^2\theta} - 1 \\ &= -1 \end{aligned}}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{-1 + \tan\theta + \sec\theta}$$

$$= \frac{\cancel{(\tan\theta + \sec\theta - 1)}(\tan\theta + \sec\theta)}{\cancel{(\tan\theta + \sec\theta - 1)}}$$

$$= \tan\theta + \sec\theta$$

$$= R.H.S.$$

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Q19:- L.H.S.

$$= \frac{1}{\csc\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\csc\theta + \cot\theta}$$

$$\frac{1}{\csc\theta - \cot\theta} + \frac{1}{\csc\theta + \cot\theta} = \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$$

$$\frac{\csc\theta + \cancel{\cot\theta} + \csc\theta - \cancel{\cot\theta}}{(\csc\theta - \cot\theta)(\csc\theta + \cot\theta)} = \frac{2}{\sin\theta}$$

$$\frac{2\csc\theta}{\csc^2\theta - \cot^2\theta} = 2\csc\theta$$

$$\frac{2\csc\theta}{1} = 2\csc\theta$$

$$2\csc\theta = 2\csc\theta$$

Hence Proved

$$= \csc^2\theta - \cot^2\theta$$

$$= \cancel{\cot^2\theta} + 1 - \cancel{\cot^2\theta}$$

$$= 1$$

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Q20:- L.H.S.

$\sin^3\theta - \cos^3\theta$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$

$= (\sin\theta - \cos\theta)(\underline{\sin^2\theta + \cos^2\theta} + \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$ $\because \sin^2\theta + \cos^2\theta = 1$

$=$ R.H.S.

Q21:- L.H.S.

$= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + \sin^2\theta\cos^2\theta + (\cos^2\theta)^2]$

$= (\sin^2\theta - \cos^2\theta)[\underline{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta} - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)((1)^2 - \sin^2\theta\cos^2\theta)$ $\because \sin^2\theta + \cos^2\theta = 1$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S.

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Q.22.- L.H.S.

$= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2]$

$= (1)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta]$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= (1)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

$=$ R.H.S.

Q.23.- L.H.S.

$= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{(1)^2 - (\sin\theta)^2}$

$= \dfrac{2}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$ $\qquad \because 1-\sin^2\theta = \cos^2\theta$

$= 2\sec^2\theta \qquad \because \frac{1}{\cos\theta} = \sec\theta$

$= R.H.S.$

Q.24:- L.H.S

$$= \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{\cos^2\theta + \sin^2\theta + \cancel{2\cos\theta\sin\theta} + \cos^2\theta + \sin^2\theta - \cancel{2\cos\theta\sin\theta}}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\cos^2\theta + 2\sin^2\theta}{1 - \sin^2\theta - \sin^2\theta} \qquad \because \cos^2\theta = 1 - \sin^2\theta$$

$$= \frac{2(\cos^2\theta + \sin^2\theta)}{1 - 2\sin^2\theta}$$

$$= \frac{2(1)}{1 - 2\sin^2\theta} \qquad \because \cos^2\theta + \sin^2\theta = 1$$

$$= \frac{2}{1 - 2\sin^2\theta}$$

$= R.H.S.$