# Mathematics

## Chapter # 01

## Number Systems

### Rational Number

Rational number is a number which can be put in the form $\frac{p}{q}$ where $p, q \in Z \wedge q \neq 0$.

**E.g**

$\sqrt{16}$, $\frac{5}{4}$, $2.7$ are rational numbers.

### Decimal representation of Rational Numbers

**1→**

### Terminating Decimals

A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal.

Every terminating decimal represents a rational number

**E.g** 202.42 , 0.00415 , 1000.41237895

02 →

## Recurring Decimals:

This is another Type of rational numbers. In general a recurring or periodic decimal is a decimal in which one or more digits repeats indefinitely

## Irrational Numbers

Irrational numbers are Those numbers which cannot be put in The form $\frac{p}{q}$, where $p, q \in Z$ and $q \neq 0$

**E.g** The numbers $\sqrt{2}$, $\sqrt{3}$, $\frac{7}{\sqrt{5}}$, $\sqrt{\frac{6}{16}}$ are irrational numbers.

## Decimal Representation of Irrational Numbers

A non-Terminating, non

recurring decimal is a decimal which neither terminates nor it is a recurring.

Thus, a non terminating, non recurring decimal represents an irrational numbers.

## Binary Operation

A binary operation is defined as a function from $A \times A$. A binary operation in a set $A$ is rule usually denoted by * That assigns To any pair of elements of A. taken in a definite order, Another element of A.

# Properties of Real Numbers

## Addition laws

### Closure law of Addition

$\forall\ a, b \in \mathbb{R},\ a+b \in \mathbb{R}$

($\forall$ stands for "for All")

### Associative law of Addition

$\forall\ a, b, c \in \mathbb{R},\ a+(b+c) = (a+b)+c$

### Additive Identity

$\forall\ a \in \mathbb{R},\ \exists 0 \in \mathbb{R}$ such That

$a+0 = 0+a = a$

($\exists$ stands for "there exists")

### Additive Inverse

$\forall\ a \in \mathbb{R},\ \exists(-a) \in \mathbb{R}$ such That

$a+(-a) = 0 = (-a)+a$

### Commutative law for Addition

$\forall\ a, b \in \mathbb{R},\ a+b = b+a$

# Multiplicative laws

## Closure Property

$\forall\ a, b \in \mathbb{R}, a \cdot b \in \mathbb{R}$

## Associative Law

$\forall\ a, b, c \in \mathbb{R}$

$a(bc) = (ab) \cdot c$

## Multiplicative Identity

$\forall\ a \in \mathbb{R}, \exists\ 1 \in \mathbb{R}$ such That

$a \cdot 1 = 1 \cdot a = a$

1 is called multiplicative identity of real numbers

## Multiplicative Inverse

$\forall\ a (\neq 0) \in \mathbb{R}$,

$\exists\ a^{-1} \in \mathbb{R}$ such that $a \cdot a^{-1} = a^{-1} \cdot a = 1$

($a^{-1}$ is also written as $\frac{1}{a}$)

## Commutative law

$\forall\ a, b \in \mathbb{R}, ab = ba$

# Multiplication-Addition law

$\forall\ a, b, c \in \mathbb{R}$

$a(b+c) = ab + ac$

$(a+b)c = ac + bc$

(Distributivity of multiplication law over addition)

# Properties of Equality

## Reflexive Property

$\forall\ a \in \mathbb{R},\ a = a$

## Symmetric Property

$\forall\ a, b \in \mathbb{R},\ a = b \Rightarrow b = a$

## Transitive Property

$\forall\ a, b, c \in \mathbb{R},\ a = b \wedge b = c \Rightarrow a = c$

## Additive Property

$\forall\ a, b, c \in \mathbb{R}$

$a = b \Rightarrow a + c = b + c$

## Multiplicative Property

$\forall,\ a, b, c \in \mathbb{R} \quad a = b$

$\Rightarrow ac = bc \wedge ca = cb$

## Cancellation Property w.r.t Addition

$\forall\ a, b, c \in \mathbb{R}$

$a+c = b+c \Rightarrow a = b$

## Cancellation Property w.r.t Multiplication

$\forall\ a, b, c \in \mathbb{R}$

$ac = bc \Rightarrow a = b,\ c \neq 0$

# Properties of Inequalities

## Trichotomy Property

$\forall\ a, b \in \mathbb{R}$

either $a = b$ or $a > b$ or $a < b$

## Transitive Property

$\forall\ a, b, c \in \mathbb{R}$

(a) $a > b \wedge b > c \Rightarrow a > c$

(b) $a < b \wedge b < c \Rightarrow a < c$

## Additive Property

$\forall\ a, b, c \in \mathbb{R}$

(a) i) $a > b \Rightarrow a+c > b+c$

ii) $a < b \Rightarrow a+c < b+c$

(b)

i) $a > b \wedge c > d \Rightarrow a+c > b+d$

ii) $a < b \wedge c < d \Rightarrow a+c < b+d$

# Multiplicative Property

(a)

$\forall\ a, b, c \in \mathbb{R}$ and $c > 0$

i →

$a > b \Rightarrow ac > bc$

ii →

$a < b \Rightarrow ac < bc$

(b)

$\forall\ a, b, c \in \mathbb{R}$ and $c < 0$

i →

$a > b \Rightarrow ac < bc$

ii →

$a < b \Rightarrow ac > bc$

(c)

$\forall\ a, b, c, d \in \mathbb{R}$ and $a, b, c, d$ are all +ve.

i →

$a > b \wedge c > d \Rightarrow ac > bd$

ii →

$a < b \wedge c < d \Rightarrow ac < bd$

# Complex Numbers

The numbers of the form $x+yi$, where $x, y \in \mathbb{R}$ and $i=\sqrt{-1}$ are called complex numbers, here $x$ is called **real part** and $y$ is called **imaginary part** of the complex number

**Examples**

$3+4i$, $2-\frac{5}{7}i$ etc are complex numbers

- Every real number is a complex number with 0 as its imaginary part.
- $\sqrt{-1}$ does not belong to the set of real numbers. we, therefore for convenience call it imaginary number and denote it by **$i$**
- The product of a real number and $i$ is also an imaginary number.