Exercise # 1.1

Question : 01

Which of the following sets have closure property w.r.t addition & multiplication

(i)

{0}

Solution

W.r.t "+"

$= \{0\}$

$\Rightarrow 0+0 = 0 \in \{0\}$

Hence, the given set is closed w.r.t addition

W.r.t "x"

$= \{0\}$

$0 \times 0 = 0 \in \{0\}$

Hence, the given set is closed w.r.t multiplication.

(ii)

{1}

Solution

w.r.t "+"

$= \{1\}$

$1+1 = 2 \notin \{1\}$

So, The given set is not closed w.r.t "+"

w.r.t "x"

$= \{1\}$

$1 \times 1 = 1 \in \{1\}$

So, The given set is closed w.r.t "x"

(iii)

{0, -1}

Solution

w.r.t "+"

$= \{0, -1\}$

$0+0 = 0 \in \{0, -1\}$

$0+(-1) = -1 \in \{0, -1\}$

$-1+(-1) = -2 \notin \{0, -1\}$

So, The given set is not closed w.r.t "+"

w.r.t "x"

$= \{0, -1\}$

$0 \times 0 = 0 \in \{0, -1\}$

$0 \times (-1) = 0 \in \{0, -1\}$

$-1 \times (-1) = 1 \notin \{0, -1\}$

So, The given set is not closed w.r.t "x"

(iv)

$\{1, -1\}$

Solution

w.r.t "+"

$= \{1, -1\}$

$1 + 1 = 2 \notin \{1, -1\}$

$1 + (-1) = 0 \notin \{1, -1\}$

$-1 + (-1) = -2 \notin \{1, -1\}$

So, The given set is not closed w.r.t "+"

w.r.t "x"

$= \{1, -1\}$

$1 \times 1 = 1 \in \{1, -1\}$

$1 \times (-1) = -1 \in \{1, -1\}$

$-1 \times (-1) = 1 \in \{1, -1\}$

So, The given set is closed w.r.t "x"

Question :02

Name the properties used in the following equations

(i)

$4+9=9+4$

Solution

$4+9=9+4$

commutative property w.r.t "+"

(ii)

$(a+1)+\frac{3}{4}=a+(1+\frac{3}{4})$

Solution

$(a+1)+\frac{3}{4}=a+(1+\frac{3}{4})$

Associative property w.r.t "+"

(iii)

$(\sqrt{3}+\sqrt{5})+\sqrt{7}=\sqrt{3}+(\sqrt{5}+\sqrt{7})$

Solution

$= (\sqrt{3}+\sqrt{5})+\sqrt{7} = \sqrt{3}+(\sqrt{5}+\sqrt{7})$

Associative property w.r.t "+"

(iv)

100+0 = 100

Solution

100+0 = 100

Additive identity

(v)

1000x 1 = 1000

Solution

1000x 1 = 1000

Multiplicative identity

(vi)

4.1 + (-4.1) = 0

Solution

4.1 + (-4.1) = 0

Additive inverse

(vii)

$a - a = 0$

Solution

$a - a = 0$

$a + (-a) = 0$

Additive inverse

(viii)

$\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$

Solution

$\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$

commutative property w.r.t "x"

(ix)

$a(b-c) = ab - ac$

Solution

$a(b-c) = ab - ac$

Left distributive property

(x)

$(x-y)z = xz - yz$

Solution

$(x-y)z = xz - yz$

Right distributive property.

مثال

$4 \times (5 \times 8) = (4 \times 5) \times 8$

Solution

$4 \times (5 \times 8) = (4 \times 5) \times 8$

Associative property w.r.t "x"

مثال

$a(b+c-d) = ab + ac - ad$

Solution

$a(b+c-d) = ab + ac - ad$

Left distributive property.

Question : 03

Name properties used in following inequalities

(i)

$-3 < -2 \Rightarrow 0 < 1$

Solution

$-3 < -2 \Rightarrow 0 < 1$

$-3 + 3 < -2 + 3 \Rightarrow 0 < 1$

Additive property

(ii)

$-5 < -4 \Rightarrow 20 > 16$

Solution

$-5 < -4 \Rightarrow 20 > 16$

$-5 \times (-4) > -4 \times (-4) \Rightarrow 20 > 16$

Multiplicative Property

(iii)

$1 > -1 \Rightarrow -3 > -5$

Solution

$1 > -1 \Rightarrow -3 > -5$

$1-4 > -1-4 \Rightarrow -3 > -5$

Additive property

(iv)

$a < 0 \Rightarrow -a > 0$

Solution

$a < 0 \Rightarrow -a > 0$

$a \times (-1) > 0 \times (-1) \Rightarrow -a > 0$

Multiplicative property

(v)

$a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

Solution

$a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

$a \times \frac{1}{ab} > b \times \frac{1}{ab} \Rightarrow \frac{1}{a} < \frac{1}{b}$

Multiplicative property

(vi)

$a > b \Rightarrow -a < -b$

Solution

$a(-1) > b(-1) \Rightarrow -a < -b$

Multiplicative property

Question : 04

Prove the following rules of addition

(i)

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Solution

L.H.S

$$= \frac{a}{c} + \frac{b}{c}$$

$$\because \frac{a}{b} = a \cdot \frac{1}{b}$$

$$= a \cdot \frac{1}{c} + b \cdot \frac{1}{c}$$

$$= (a+b) \cdot \frac{1}{c} \quad \text{(distributive property)}$$

$$= \frac{a+b}{c} \qquad \because a \cdot \frac{1}{c} = \frac{a}{c}$$

$$= R.H.S$$

Hence proved

(ii)

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Solution

L.H.S

$$= \frac{a}{b} + \frac{c}{d}$$

$$= \frac{a}{b} \cdot 1 + \frac{c}{d} \cdot 1 \quad \text{(multiplicative identity)}$$

$$= \frac{a}{b}\left(\frac{d}{d}\right) + \frac{c}{d}\left(\frac{b}{b}\right) \quad \text{(Multiplicative inverse)}$$

$\because \frac{d}{d} = d \cdot \frac{1}{d}, \frac{b}{b} = b \cdot \frac{1}{b}$

$$= \frac{ad}{bd} + \frac{cb}{db}$$

$$= \frac{ad}{bd} + \frac{bc}{bd} \quad \text{(commutative property w.r.t "x")}$$

$$= ad \cdot \frac{1}{bd} + bc \cdot \frac{1}{bd}$$

$$= (ad + bc) \cdot \frac{1}{bd} \quad \text{(Distributive property)}$$

$$= \frac{ad+bc}{bd} \quad \because a \cdot \frac{1}{b} = \frac{a}{b}$$

$$= \text{R.H.S}$$

Question : 05

Prove that

$$\frac{-7}{12} - \frac{5}{18} = \frac{-21-10}{36}$$

Solution

L.H.S

$$= \frac{-7}{12} - \frac{5}{18}$$

$$= \frac{-7}{12}\left(\frac{3 \cdot 1}{3}\right) - \frac{5}{18}\left(\frac{2 \cdot 1}{2}\right)$$

(Multiplicative inverse)

$$= \frac{-7}{12} \cdot \frac{3}{3} - \frac{5}{18} \cdot \frac{2}{2}$$

$\because a \cdot \frac{1}{b} = \frac{a}{b}$

$$= \frac{-21}{36} - \frac{10}{36}$$

$\because \frac{a}{b} = a \cdot \frac{1}{b}$

$$= \frac{-21 \cdot 1}{36} - \frac{10 \cdot 1}{36}$$

$$= \frac{(-21-10) \cdot 1}{36}$$ (Distributive property)

$$= \frac{-21-10}{36}$$

= R.H.S

Hence proved

Question: 06

Simplify by justifying each step

(i)

$$\frac{4+16x}{4}$$

Solution

$$= \frac{4+16x}{4}$$

$$= (4+16x) \cdot \frac{1}{4} \quad \because \frac{a}{b} = a \cdot \frac{1}{b}$$

$$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16x \quad \text{(distributive property)}$$

$$= \frac{1}{4} \cdot 4 + 4x \quad \text{(Multiplicative inverse)}$$

$$= 1 + 1 \cdot 4x \quad \text{(multiplicative identity)}$$

$$= 1 + 4x$$

(ii)

$$\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$$

Solution

$$= \left(\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}\right) \cdot 1 \quad \text{(Multiplicative identity)}$$

$$= \left(\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}\right) \cdot 20 \cdot \frac{1}{20} \quad \text{(Multiplicative inverse)}$$

$$= \frac{\left(\frac{1}{4}+\frac{1}{5}\right)\cdot 20}{\left(\frac{1}{4}-\frac{1}{5}\right)\cdot 20} \qquad \because 20 \cdot \frac{1}{20} = \frac{20}{20}$$

$$= \frac{20\cdot\frac{1}{4}+20\cdot\frac{1}{5}}{20\cdot\frac{1}{4}-20\cdot\frac{1}{5}} \quad \text{(Distributive Property)}$$

$$= \frac{\left(4\cdot\frac{1}{4}\right)5+\left(5\cdot\frac{1}{5}\right)4}{\left(4\cdot\frac{1}{4}\right)5-\left(5\cdot\frac{1}{5}\right)4} \quad \text{(Multiplicative inverse)}$$

$$= \frac{1\times 5+1\times 4}{1\times 5-1\times 4} \quad \text{(Multiplicative identity)}$$

$$= \frac{5+4}{5-4}$$

$$= \frac{9}{1}$$

(iii)

$$\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}}$$

Solution

$$= \frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}}$$

$$= \left(\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}}\right) \cdot 1 \quad \text{(Multiplicative identity)}$$

$$= \left(\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}}\right) \cdot bd \cdot \frac{1}{bd} \quad \text{(Multiplicative inverse)}$$

$$\because bd \cdot \frac{1}{bd} = \frac{bd}{bd}$$

$$= \frac{\left(\frac{a}{b} + \frac{c}{d}\right) \cdot bd}{\left(\frac{a}{b} - \frac{c}{d}\right) \cdot bd}$$

$$= \frac{a \cdot \frac{1}{b} \cdot b \cdot d + c \cdot \frac{1}{d} \cdot b \cdot d}{a \cdot \frac{1}{b} \cdot b \cdot d - c \cdot \frac{1}{d} \cdot b \cdot d} \quad \text{(Distributive property)}$$

$$= \frac{a \cdot \left(b \cdot \frac{1}{b}\right) \cdot d + c\left(d \cdot \frac{1}{d}\right) \cdot b}{a \cdot \left(b \cdot \frac{1}{b}\right) \cdot d - c\left(d \cdot \frac{1}{d}\right) \cdot b} \quad \text{(Multiplicative inverse)}$$

$$= \frac{a \cdot 1 \cdot d + c \cdot 1 \cdot b}{a \cdot 1 \cdot d - c \cdot 1 \cdot b} \quad \text{(Multiplicative identity)}$$

$$= \frac{ad + cb}{ad - cb}$$