Exercise 1.2

Question : 04

Simplify the following

(i)

i^9

Solution

$= i^9$

$= i \times i^8$

$= i \times (i^2)^4 \qquad \because i^2 = -1$

$= i \times (-1)^4$

$= i$

(ii)

i^{14}

Solution

$= i^{14}$

$= (i^2)^7 \qquad \because i^2 = -1$

$= (-1)^7$

$= -1$

(iii)

$(-i)^{19}$

Solution

$= (-i)^{19}$

$= (-1)^{19} \cdot \times (i)^{19}$

$= -1 \times i \times i^{18}$ $\quad \because i^2 = -1$

$= -i \times (i^2)^9$

$= -i \times (-1)^9$

$= -i \times (-1)$

$= i$

(iv)

$(-1)^{-\frac{21}{2}}$

Solution

$= (-1)^{-\frac{21}{2}}$

$= \left((-1)^{1/2}\right)^{-21}$

$\because i^2 = -1$

$= \left(i^{2 \cdot \frac{1}{2}}\right)^{-21}$

$= (i)^{-21} = \dfrac{1}{i^{21}} = \dfrac{1}{i^{20} \cdot i}$

$= \dfrac{1}{(i^2)^{10} \cdot i} = \dfrac{1}{(-1)^{10} \cdot i} = \dfrac{1}{1 \cdot i} = \dfrac{1}{i}$

$= \dfrac{1}{i} \times \dfrac{i}{i} = \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

Question : 05

Write in terms of i

(i)

$\sqrt{-1}\, b$

Solution

$= \sqrt{-1}\, b$

$= bi \qquad \because i = \sqrt{-1}$

(ii)

$\sqrt{-5}$

Solution

$= \sqrt{-5}$

$= \sqrt{-1} \times \sqrt{5} \qquad \because i = \sqrt{-1}$

$= \sqrt{5}\, i$

(iii)

$\sqrt{\dfrac{-16}{25}}$

Solution

$$= \sqrt{\frac{-16}{25}}$$

$$= \sqrt{-1} \cdot \sqrt{\frac{16}{25}} \qquad \because i = \sqrt{-1}$$

$$= \frac{4}{5} i$$

(iv)

$$\sqrt{\frac{1}{-4}}$$

Solution

$$= \sqrt{\frac{1}{-4}}$$

$$= \sqrt{\frac{1}{4}} \cdot \sqrt{-1} \qquad \because i = \sqrt{-1}$$

$$= \frac{1}{2} i$$

Simplify the following

Question: 06

$(7,9)+(3,-5)$

solution

$= (7,9)+(3,-5)$

$= 7+9i+3-5i$

$= 10+4i$

$= (10,4)$

Question : 07

$(8,-5)-(-7,4)$

Solution

$= (8,-5)-(-7,4)$

$= 8-5i-(-7+4i)$

$= 8-5i+7i-4i$

$= 15-9i$

$= (15,-9)$

Question : 08

$(2,6)(3,7)$

Solution

$= (2,6)(3,7)$

$= (2+6i)(3+7i)$

$= 6+14i+18i+42i^2$

$= 6+32i+42(-1) \quad \because i^2=-1$

$= 6+32i-42$

$= 32i-36$

$= (-36,32)$

Question : 09

$(5,-4)(-3,-2)$

solution

$= (5,-4)(-3,-2)$

$= (5-4i)(-3-2i)$

$= -15-10i+12i+8i^2 \quad \because i^2=-1$

$= -15+2i+8(-1)$

$= -15+2i-8$

$= -23+2i$

$= (-23,2)$

Question : 10

$(0,3)(0,5)$

Solution

$= (0,3)(0,5)$

$= (0+3i)(0+5i)$

$= 0+0+0+15i^2 \quad \because i^2 = -1$

$= 15(-1)$

$= -15$

Question : 11

$(2,6) \div (3,7)$

Solution

$= (2,6) \div (3,7)$

$= \dfrac{2+6i}{3+7i}$

$= \dfrac{2+6i}{3+7i} \times \dfrac{3-7i}{3-7i}$

$= \dfrac{(2+6i)(3-7i)}{(3+7i)(3-7i)}$

$\because (a+b)(a-b) = a^2 - b^2$

$= \dfrac{6-14i+18i-42i^2}{(3)^2-(7i)^2}$

$$= \frac{6+4i-42(-1)}{9-49i^2}$$

$$= \frac{6+4i+42}{9-49(-1)}$$

$$= \frac{48+4i}{9+49}$$

$$= \frac{48+4i}{58}$$

$$= \frac{48}{58} + \frac{4i}{58}$$

$$= \frac{24}{29} + \frac{2i}{29}$$

$$= \left(\frac{24}{29}, \frac{2}{29}\right)$$

Question : 12

$(5,-4) \div (-3,-8)$

solution

$$= (5,-4) \div (-3,-8)$$

$$= (5-4i) \div (-3-8i)$$

$$= \frac{5-4i}{-3-8i} \times \frac{-3+8i}{-3+8i}$$

$$= \frac{(5-4i)(-3+8i)}{(-3-8i)(-3+8i)}$$

$\because (a-b)(a+b) = a^2 - b^2$

$$= \frac{-15 + 40i + 12i - 32i^2}{(-3)^2 - (8i)^2}$$

$$= \frac{-15 + 52i - 32(-1)}{9 - 64i^2}$$

$$= \frac{-15 + 52i + 32}{9 - 64(-1)}$$

$$= \frac{17 + 52i}{9 + 64}$$

$$= \frac{17 + 52i}{73}$$

$$= \frac{17}{73} + \frac{52}{73}i$$

$$= \left(\frac{17}{73}, \frac{52}{73}\right)$$

Question : 13

prove that the sum as well as product of any two complex conjugate number is a real number

Solution

Let

$z = a + bi$ is a complex number

$\bar{z} = \overline{a+bi}$

$\bar{z} = a - bi$

Sum

$z + \bar{z} = a + \cancel{bi} + a - \cancel{bi}$

$z + \bar{z} = 2a$ is a real number

Product

$z \cdot \bar{z} = (a+bi)(a-bi)$

$z \cdot \bar{z} = a^2 - \cancel{abi} + \cancel{abi} - b^2 i^2$

$z \cdot \bar{z} = a^2 - b^2(-1)$

$z \cdot \bar{z} = a^2 + b^2$ is a real number

Hence

Sum as well as product of two conjugate complex number is a real number.

Question : 14

Find the multiplicative inverse of each of the following numbers

(i)

$(-4, 7)$

Solution

$= (-4, 7)$

$z = -4 + 7i$

$\frac{1}{z} = \frac{1}{-4+7i}$

$= \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$

$= \frac{-4-7i}{(-4+7i)(-4-7i)}$

$(a+b)(a-b) = a^2 - b^2$

$= \frac{-4-7i}{(-4)^2 - (7i)^2}$

$= \frac{-4-7i}{16 - 49i^2}$

$= \frac{-4-7i}{16 - 49(-1)}$

$$= \frac{-4-7i}{16+49}$$

$$= \frac{-4-7i}{65}$$

$$= \frac{-4}{65} - \frac{7}{65}i$$

$$= \left(\frac{-4}{65}, \frac{-7}{65}\right)$$

(ii)

$(\sqrt{2}, -\sqrt{5})$

Solution

$$= (\sqrt{2}, -\sqrt{5})$$

let

$$z = \sqrt{2} - \sqrt{5}i$$

$$\frac{1}{z} = \frac{1}{\sqrt{2}-\sqrt{5}i}$$

$$= \frac{1}{\sqrt{2}-\sqrt{5}i} \times \frac{\sqrt{2}+\sqrt{5}i}{\sqrt{2}+\sqrt{5}i}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{(\sqrt{2}-\sqrt{5}i)(\sqrt{2}+\sqrt{5}i)}$$

$(a-b)(a+b) = a^2-b^2$

$$= \frac{\sqrt{2}+\sqrt{5}i}{(\sqrt{2})^2-(\sqrt{5}i)^2}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{2-5i^2}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{2-5(-1)}$$

$$= \frac{\sqrt{2}+\sqrt{6}i}{2+5} \quad \because i^2=-1$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{7}$$

$$= \frac{\sqrt{2}}{7}+\frac{\sqrt{5}}{7}i$$

$$= \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$$

Q iii

$(1, 0)$

Solution

$= (1, 0)$

Let

$z = 1+0i$

$$\frac{1}{z} = \frac{1}{1+0i} \times \frac{1-0i}{1-0i}$$

$$= \frac{1-0i}{1^2-(0i)^2}$$

$$= \frac{1-0i}{1} = 1-0i$$

Question : 15

Factorize the following

(i)

a^2+4b^2

Solution

$= a^2+4b^2$

$= a^2-(-4b^2)$

$= a^2-(2bi)^2$

$\because a^2-b^2=(a+b)(a-b)$

$= (a-2bi)(a+2bi)$

(ii)

$9a^2+16b^2$

Solution

$= 9a^2+16b^2$

$= 9a^2-(-16b^2)$

$= (3a)^2-(4bi)^2$

$$a^2-b^2=(a+b)(a-b)$$
$$=(3a+4bi)(3a-4bi)$$

(iii)

$3x^2+3y^2$

Solution

$$=3x^2+3y^2$$
$$=3(x^2+y^2)$$
$$=3(x^2-(-y^2))$$
$$=3((x)^2-(yi)^2)$$
$$\because a^2-b^2=(a-b)(a+b)$$
$$=3(x-yi)(x+yi)$$

Question : 16

Seperate into real and imaginary parts

(i)

$$\frac{2-7i}{4+5i}$$

Solution

$$= \frac{2-7i}{4+5i}$$

$$= \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{(2-7i)(4-5i)}{(4+5i)(4-5i)}$$

$\because (a+b)(a-b) = a^2-b^2$

$$= \frac{8-10i-28i+35i^2}{(4)^2-(5i)^2}$$

$\because i^2 = -1$

$$= \frac{8-38i+35(-1)}{16-25(-1)}$$

$$= \frac{8-38i-35}{16+25}$$

$$= \frac{-27-38i}{41}$$

$$= \frac{-27}{41} - \frac{38}{41}i$$

Q iii

$$\frac{(-2+3i)^2}{1+i}$$

Solution

$$= \frac{(-2+3i)^2}{1+i}$$

$$= \frac{(-2+3i)^2}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{(-2+3i)^2 \times (1-i)}{(1+i)(1-i)}$$

$$= \frac{(4+9i^2-12i)(1-i)}{(1+i)(1-i)}$$

$$\because a^2 - b^2 = (a+b)(a-b)$$

$$= \frac{(4+9(-1)-12i)(1-i)}{1^2 - i^2}$$

$$= \frac{(4-9-12i)(1-i)}{1-(-1)} \quad \because i^2 = -1$$

$$= \frac{(-5-12i)(1-i)}{1+1}$$

$$= \frac{-5+5i-12i+12i^2}{2}$$

$$= \frac{-5-7i+12(-1)}{2}$$

$$= \frac{-5-7i-12}{2}$$

$$= \frac{-17-7i}{2}$$

$$= \frac{-17}{2} - \frac{7}{2}i$$

(iii)

$$\frac{i}{1+i}$$

Solution

$$= \frac{i}{1+i}$$

$$= \frac{i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{i(1-i)}{(1+i)(1-i)}$$

$\because i^2 = -1$

$$= \frac{i-i^2}{1-i^2}$$

$$= \frac{i-(-1)}{1-(-1)}$$

$$= \frac{i+1}{1+1}$$

$$= \frac{1+i}{2}$$

$$= \frac{1}{2} + \frac{1}{2}i$$