Chapter 3

Matrix

Ex # 3.1

Q1:- $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix}$

i) $4A - 3A = A$

L.H.S.

$= 4A - 3A$

$= 4\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 8 & 12 \\ 4 & 20 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$= A$

$=$ R.H.S.

ii) $3B - 3A = 3(B - A)$

L.H.S.

$$= 3B - 3A$$

$$= 3\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - 3\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 21 \\ 18 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix}$$

R.H.S.

$$= 3(B - A)$$

$$= 3\left(\begin{bmatrix} 1 & 7 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}\right)$$

$$= 3\begin{bmatrix} -1 & 4 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 12 \\ 15 & -3 \end{bmatrix}$$

L.H.S = R.H.S.

Q.2.- $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, $A^4 = I_2$

$A^4 = A^2 A^2$

$A^2 = AA$

$= \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$

$= \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^4 = A^2 A^2$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^4 = I_2$

Hence Proved!

Q3:-

i) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$x+3=2$, $3y-4=2$

$x=2-3$, $3y=2+4$

$\boxed{x=-1}$, $3y=6$

$y=\frac{6}{3}$

$\boxed{y=2}$

ii) $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$

$x+3=y$ — 1 ; $3y-4=2x$ — 2

Put $y=x+3$ in 2

$3(x+3)-4=2x$

$3x+9-4=2x$

$3x+5=2x$

$$3x - 2x = -5$$

$$\boxed{x = -5}$$

Put $x = -5$ in ①

$$y = x + 3$$

$$y = -5 + 3$$

$$\boxed{y = -2}$$

Q4 soln

i) $4A-3B$

$$= 4\begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} - 3\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 8 & 12 \\ 4 & 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 9 & 6 \\ 3 & -3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -1 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

ii) $A+3(B-A)$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\left(\begin{bmatrix} 0 & 3 & 2 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -3 \\ 0 & -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 0 \\ 1 & -3 & 2 \end{bmatrix}$$

Q5:- Find x & y

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & x & y \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & x \\ 1 & y & 3 \end{bmatrix} + \begin{bmatrix} 2 & 2x & 2y \\ 0 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2x & x+2y \\ 1 & y+4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 6 & 1 \end{bmatrix}$$

$2x = -2$, $y+4 = 6$

$x = \frac{-2}{2}$, $y = 6-4$

$\boxed{x = -1}$, $\boxed{y = 2}$

Q6:- $A = [a_{ij}]_{3\times3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

i) $\lambda(\mu A) = (\lambda\mu)A$

L.H.S.

$$= \lambda(\mu A)$$

$$= \lambda\left(\mu \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right)$$

$$= \lambda \begin{bmatrix} \mu a_{11} & \mu a_{12} & \mu a_{13} \\ \mu a_{21} & \mu a_{22} & \mu a_{23} \\ \mu a_{31} & \mu a_{32} & \mu a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda\mu a_{11} & \lambda\mu a_{12} & \lambda\mu a_{13} \\ \lambda\mu a_{21} & \lambda\mu a_{22} & \lambda\mu a_{23} \\ \lambda\mu a_{31} & \lambda\mu a_{32} & \lambda\mu a_{33} \end{bmatrix}$$

$$= (\lambda\mu) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= (\lambda\mu) A$$

$$= R.H.S.$$

ii) $(\lambda+\mu)A = \lambda A + \mu A$

L.H.S.

$$= (\lambda+\mu)A$$

$$= (\lambda+\mu) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda+\mu)a_{11} & (\lambda+\mu)a_{12} & (\lambda+\mu)a_{13} \\ (\lambda+\mu)a_{21} & (\lambda+\mu)a_{22} & (\lambda+\mu)a_{23} \\ (\lambda+\mu)a_{31} & (\lambda+\mu)a_{32} & (\lambda+\mu)a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} + \mu a_{11} & \lambda a_{12} + \mu a_{12} & \lambda a_{13} + \mu a_{13} \\ \lambda a_{21} + \mu a_{21} & \lambda a_{22} + \mu a_{22} & \lambda a_{23} + \mu a_{23} \\ \lambda a_{31} + \mu a_{31} & \lambda a_{32} + \mu a_{32} & \lambda a_{33} + \mu a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix} + \begin{bmatrix} \mu a_{11} & \mu a_{12} & \mu a_{13} \\ \mu a_{21} & \mu a_{22} & \mu a_{23} \\ \mu a_{31} & \mu a_{32} & \mu a_{33} \end{bmatrix}$$

$$= \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \mu \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$= \lambda A + \mu A$

$= R.H.S.$

iii) $\lambda A - A = (\lambda - 1)A$

L.H.S.

$= \lambda A - A$

$$= \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{bmatrix} - \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} - a_{11} & \lambda a_{12} - a_{12} & \lambda a_{13} - a_{13} \\ \lambda a_{21} - a_{21} & \lambda a_{22} - a_{22} & \lambda a_{23} - a_{23} \\ \lambda a_{31} - a_{31} & \lambda a_{32} - a_{32} & \lambda a_{33} - a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} (\lambda - 1) a_{11} & (\lambda - 1) a_{12} & (\lambda - 1) a_{13} \\ (\lambda - 1) a_{21} & (\lambda - 1) a_{22} & (\lambda - 1) a_{23} \\ (\lambda - 1) a_{31} & (\lambda - 1) a_{23} & (\lambda - 1) a_{33} \end{bmatrix}$$

$$= (\lambda - 1) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$= (\lambda - 1) A$$

$= R.H.S$

Q.7.

$$A = [a_{ij}]_{2\times 3} \quad , B = [b_{ij}]_{2\times 3}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$\lambda(A+B) = \lambda A + \lambda B$$

L.H.S.

$$= \lambda(A+B)$$

$$= \lambda\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}\right)$$

$$= \lambda \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11}+\lambda b_{11} & \lambda a_{12}+\lambda b_{12} & \lambda a_{13}+\lambda b_{13} \\ \lambda a_{21}+\lambda b_{21} & \lambda a_{22}+\lambda b_{22} & \lambda a_{23}+\lambda b_{23} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \end{bmatrix} + \begin{bmatrix} \lambda b_{11} & \lambda b_{12} & \lambda b_{13} \\ \lambda b_{21} & \lambda b_{22} & \lambda b_{23} \end{bmatrix}$$

$$= \lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} + \lambda \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$$

$$= \lambda A + \lambda B$$

$$= R.H.S.$$

Q8:- $A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A^2 = AA$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix}$$

$1+2a=0$, $2+2b=0$

$2a=-1$, $2b=-2$

$\boxed{a = -\frac{1}{2}}$, $b = \frac{-2}{2}$

$$\boxed{b=-1}$$

Q8. $A = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$, $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Find a, b

$A^2 = A\,A$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix} \begin{bmatrix} 1 & -1 \\ a & b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-a & -1-b \\ a+ab & -a+b^2 \end{bmatrix}$$

$1-a=1$, $-1-b=0$

$-a=1-1$, $-b=1$

$-a=0$, $\boxed{b=-1}$

$\Rightarrow \boxed{a=0}$,

Q10:- $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$(A+B)^t = A^t + B^t$

L.H.S.

$= (A+B)^t$

$A+B = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

$A+B = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 5 & 0 \end{bmatrix}$

$(A+B)^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$

R.H.S $= A^t + B^t$

$A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$, $B^t = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$

$A^t + B^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$

$$A^t + B^t = \begin{bmatrix} 3 & 1 \\ 2 & 5 \\ 2 & 0 \end{bmatrix}$$

L.H.S. = R.H.S.

Q11:- $A^3 = ?$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$A^3 = A^2 A$

$A^2 = A A$

$$A^2 = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It is a null matrix.

Q12:

i) $X \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix}$

Let

$$X A = B$$

$$X = BA^{-1} \text{ ——— } (1)$$

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$$

$$|A| = (5)(1) - (2)(-2)$$
$$= 5 + 4$$
$$= 9 \neq 0$$

$$Adj\,A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj\,A$$

$$= \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

By putting the values in ①

$$X = \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} -1 & 5 \\ 12 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$X = \frac{1}{9}\begin{bmatrix} -1+10 & 2+25 \\ 12+6 & -24+15 \end{bmatrix}$$

$$X = \frac{1}{9}\begin{bmatrix} 9 & 27 \\ 18 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{9}{9} & \frac{27}{9} \\ \frac{18}{9} & \frac{9}{9} \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

Q/20- ii) $\begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix} X = \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$

let $A \, X = B$

$X = A^{-1}B$ — (1)

$$|A| = \begin{vmatrix} 5 & 2 \\ -2 & 1 \end{vmatrix}$$

$= (5)(1) - (2)(-2)$

$= 5+4$

$= 9 \neq 0$

$$\text{Adj} A = \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A$$

$$A^{-1} = \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix}$$

By putting the values in (1)

$$X = \frac{1}{9} \begin{bmatrix} 1 & -2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 10 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 2-10 & 1-20 \\ 4+25 & 2+50 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} -8 & -19 \\ 29 & 52 \end{bmatrix}$$

$$= \begin{bmatrix} -8/9 & -19/9 \\ 29/9 & 52/9 \end{bmatrix}$$

Q13. i/p

i) $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 0 & 0 \\ 7 & 2 \end{bmatrix}$

$\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & -7 \\ 7 & 2 \end{bmatrix}$

Let

$B \; A = C$

$A = B^{-1} C$ —— (1)

$|B| = \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix}$

$|B| = (5)(1) - (-1)(3)$

$= 5 + 3$

$= 8 \neq 0$

$Adj B = \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} Adj B$

$= \frac{1}{8} \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}$

By putting the values in (1)

$$A = \frac{1}{8}\begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 & -7 \\ 7 & 2 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 3+7 & -7+2 \\ -9+35 & 21+10 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 10 & -5 \\ 26 & 31 \end{bmatrix}$$

$$= \begin{bmatrix} 10/8 & -5/8 \\ 26/8 & 31/8 \end{bmatrix}$$

$$= \begin{bmatrix} 5/4 & -5/8 \\ 13/4 & 31/8 \end{bmatrix}$$

Q 13.

ii) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$

let

$$BA = C$$

$$A = B^{-1}C \quad — (1)$$

$$|B| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= (2)(2) - (-1)(-1)$$

$$= 4 - 1$$

$$= 3 \neq 0$$

$$\text{Adj}B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}B$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

By putting the values in (I).

$$A = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & -3 & 8 \\ 3 & 3 & -7 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 0+3 & -6+3 & 16-7 \\ 0+6 & -3+6 & 8-14 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 & -3 & 9 \\ 6 & 3 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$$

Q14: L.H.S.

$$= \begin{bmatrix} 8\cos\phi & 0 & -\sin\phi \\ 0 & 8 & 0 \\ 8\sin\phi & 0 & \cos\phi \end{bmatrix}\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -8\sin\phi & 0 & 8\cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\cos^2\phi + 0 + r\sin^2\phi & 0+0+0 & r\cos\phi\sin\phi + 0 - r\cos\phi\sin\phi \\ 0+0+0 & 0+r+0 & 0+0+0 \\ r\sin\phi\cos\phi + 0 - r\sin\phi\cos\phi & 0+0+0 & r\sin^2\phi + 0 + r\cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r\cos^2\phi + r\sin^2\phi & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r\sin^2\phi + r\cos^2\phi \end{bmatrix}$$

$$= \begin{bmatrix} r(\cos^2\phi + \sin^2\phi) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(\sin^2\phi + \cos^2\phi) \end{bmatrix}$$

$$= \begin{bmatrix} r(1) & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r(1) \end{bmatrix}$$

$$= r \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= r I_3 = R.H.S.$$