

Ex # 3.3

Q1:-

$$\textcircled{i)} \begin{vmatrix} 5 & -2 & -4 \\ 3 & -1 & -3 \\ -2 & 1 & 2 \end{vmatrix}$$

$$= 5 \begin{vmatrix} -1 & -3 \\ 1 & 2 \end{vmatrix} - (-2) \begin{vmatrix} 3 & -3 \\ -2 & 2 \end{vmatrix} + (-4) \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix}$$

$$= 5 [(-1)(2) - (-3)(1)] + 2 [(3)(2) - (-3)(-2)] - 4 [(3)(1) - (-1)(-2)]$$

$$= 5 [-2 + 3] + 2 [6 - 6] - 4 [3 - 2]$$

$$= 5 [1] + 2 [0] - 4 [1]$$

$$= 5 + 0 - 4$$

$$= 1$$

$$iv) \begin{vmatrix} a+l & a-l & a \\ a & a+l & a-l \\ a-l & a & a+l \end{vmatrix}$$

$$= (a+l) \begin{vmatrix} a+l & a-l \\ a & a+l \end{vmatrix} - (a-l) \begin{vmatrix} a & a-l \\ a-l & a+l \end{vmatrix} + a \begin{vmatrix} a & a+l \\ a-l & a \end{vmatrix}$$

$$= (a+l)[(a+l)^2 - a(a-l)] - (a-l)[a(a+l) - (a-l)^2] + a[a^2 - (a+l)(a-l)]$$

$$= (a+l)[\cancel{a^2} + l^2 + 2al - \cancel{a^2} + al] - (a-l)[\cancel{a^2} + al - \cancel{a^2} - l^2 + 2al] + a[a^2 - \cancel{a^2} - l^2 + 2al]$$

$$+ a[a^2 - \cancel{a^2} + l^2] \leftarrow$$

$$\textcircled{a+l}$$

$$= (a+l)(l^2 + 3al) - (a-l)(3al - l^2) + al^2$$

$$= al^2 + 3a^2l + \cancel{l^3} + 3al^2 - 3a^2l + al^2 + 3al^2 - \cancel{l^3} + al^2$$

$$= 9al^2$$

$$vi) \begin{vmatrix} 2a & a & a \\ b & 2b & b \\ c & c & 2c \end{vmatrix}$$

$$= 2a \begin{vmatrix} 2b & b \\ c & 2c \end{vmatrix} - a \begin{vmatrix} b & b \\ c & 2c \end{vmatrix} + a \begin{vmatrix} b & 2b \\ c & c \end{vmatrix}$$

$$= 2a [(2b)(2c) - (b)(c)] - a [b(2c) - bc] + a [bc - 2bc]$$

$$= 2a [4bc - bc] - a [2bc - bc] + a [-bc]$$

$$= 2a [3bc] - a [bc] - abc$$

$$= 6abc - abc - abc$$

$$= 6abc - 2abc$$

$$= 4abc$$

Q.2 Without Expansion

$$\begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\begin{array}{c} \text{L.H.S.} \end{array}
 \begin{array}{ccc} C_1 & C_2 & C_3 \\ \uparrow & \uparrow & \uparrow \\ \begin{vmatrix} 6 & 7 & 8 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{vmatrix} \end{array}
 \begin{array}{l} \rightarrow R_1 \\ \rightarrow R_2 \\ \rightarrow R_3 \end{array}$$

$$= \begin{vmatrix} 6 & 7 & 1 \\ 3 & 4 & 1 \\ 2 & 3 & 1 \end{vmatrix} \quad C_3 - C_2$$

$$\begin{array}{l} 8 - 7 = 1 \\ 5 - 4 = 1 \\ 4 - 3 = 1 \end{array}$$

$$= \begin{vmatrix} 6 & 1 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix} \quad C_2 - C_1$$

Rows 2, 3 & Columns 2, 3 are identical
 Determinant is 0

∴ Answer is 0

$$= 0 \quad \because C_2 = C_3$$

= R.H.S

ii) L.H.S.

$$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 3 & -1 \\ 0 & 1 & 0 \\ 5 & -3 & 5 \end{vmatrix} \quad C_1 - C_2$$

$$= 0$$

$$\therefore C_1 = C_3$$

= R.H.S.

L.H.S.

000)
111)

1	2	3
4	5	6
7	8	9

$$= \begin{array}{|ccc|} \hline 1 & 2 & 1 \\ \hline 4 & 5 & 1 \\ \hline 7 & 8 & 1 \\ \hline \end{array} \quad C_3 - C_2$$

$$= \begin{array}{|ccc|} \hline 1 & 1 & 1 \\ \hline 4 & 1 & 1 \\ \hline 7 & 1 & 1 \\ \hline \end{array} \quad C_2 - C_1$$

$$= 0 \quad \therefore C_2 = C_3$$

= R.H.S.

Q.3:-

e) R.H.S.

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & \alpha_{13} \\ a_{21} & a_{22} & \alpha_{23} \\ a_{31} & a_{32} & \alpha_{33} \end{vmatrix}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} + \alpha_{13} \\ a_{21} & a_{22} & a_{23} + \alpha_{23} \\ a_{31} & a_{32} & a_{33} + \alpha_{33} \end{vmatrix}$$

= L.H.S.

$$\text{ex) } \begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix} = 9 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

L.H.S.

$$= \begin{vmatrix} 2 & 3 & 0 \\ 3 & 9 & 6 \\ 2 & 15 & 1 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 2 & 1 & 0 \\ 3 & 3 & 6 \\ 2 & 5 & 1 \end{vmatrix} \begin{array}{l} \text{Taking '3' as} \\ \text{common from} \\ C_2 \end{array}$$

$$= 3 \times 3 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix} \begin{array}{l} \text{Taking '3' as} \\ \text{common from} \\ R_2 \end{array}$$

$$= 9 \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 2 \\ 2 & 5 & 1 \end{vmatrix}$$

= R.H.S.

∴ (iii) L.H.S.

$$= \begin{vmatrix} a+l & a & a \\ a & a+l & a \\ a & a & a+l \end{vmatrix}$$

$$= (a+l) \begin{vmatrix} a+l & a \\ a & a+l \end{vmatrix} - a \begin{vmatrix} a & a \\ a & a+l \end{vmatrix} + a \begin{vmatrix} a & a+l \\ a & a \end{vmatrix}$$

$$= (a+l)[(a+l)^2 - a^2] - a[a(a+l) - a^2] + a[a^2 - a(a+l)]$$

$$= (a+l)(a^2+l^2+2al-a^2) - a[a^2+al-a^2] + a[a^2-a^2-al]$$

$$= (a+l)[l^2+2al] - a[al] + a[-al]$$

$$= \underline{al^2} + 2a^2l + l^3 + 2al^2 - \underline{a^2l} - \underline{a^2l}$$

$$= 3al^2 + 2a^2l - 2a^2l + l^3$$

$$= 3al^2 + l^3$$

$$= l^2(3a+l)$$

$$= R.H.S.$$

iv) L.H.S.

$$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{vmatrix}$$

$$= \frac{1}{xyz} \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ xyz & xyz & xyz \end{vmatrix} \begin{array}{l} \text{multiply } C_1 \text{ by} \\ 'x', C_2 \text{ by } 'y' \\ \& C_3 \text{ by } 'z' \end{array}$$

$$= \frac{xyz}{xyz} \left| \begin{array}{ccc|c} x & y & z & \text{Taking 'xyz' as common} \\ x^2 & y^2 & z^2 & \\ 1 & 1 & 1 & \text{from } R_3 \end{array} \right|$$

$$= (-1) \left| \begin{array}{ccc|c} 1 & 1 & 1 & R_1 \leftrightarrow R_3 \\ x^2 & y^2 & z^2 & \\ x & y & z & \end{array} \right|$$

$$= (-1)(-1) \left| \begin{array}{ccc|c} 1 & 1 & 1 & R_2 \leftrightarrow R_3 \\ x & y & z & \\ x^2 & y^2 & z^2 & \end{array} \right|$$

$$= \left| \begin{array}{ccc|c} 1 & 1 & 1 & \\ x & y & z & \\ x^2 & y^2 & z^2 & \end{array} \right|$$

$$= \text{R.H.S.}$$

v) L.H.S.

$$= \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= (b+c) \begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix} - a \begin{vmatrix} b & b \\ c & a+b \end{vmatrix} + a \begin{vmatrix} b & c+a \\ c & c \end{vmatrix}$$

$$= (b+c) [(c+a)(a+b) - bc] - a [b(a+b) - bc] + a [bc - c(c+a)]$$

$$= (b+c) [ca + \cancel{bc} + a^2 + ab - \cancel{bc}] - a [ab + b^2 - bc] + a [bc - c^2 - ca]$$

$$= \underline{abc} + \cancel{a^2b} + \cancel{ab^2} + \underline{ca^2} + \underline{abc} - \cancel{a^2b} - \cancel{ab^2} + \underline{abc} + \underline{abc} - \cancel{ca^2} - \cancel{ca^2}$$

$$= 4abc$$

= R.H.S.

v_i^o) L.H.S.

$$= \begin{vmatrix} b & -1 & a \\ a & b & 0 \\ 1 & a & b \end{vmatrix}$$

$$= b \begin{vmatrix} b & 0 \\ a & b \end{vmatrix} - (-1) \begin{vmatrix} a & 0 \\ 1 & b \end{vmatrix} + a \begin{vmatrix} a & b \\ 1 & a \end{vmatrix}$$

$$= b(b^2 - 0) + 1(ab - 0) + a(a^2 - b)$$

$$= b(b^2) + 1(ab) + a^3 - ab$$

$$= b^3 + \cancel{ab} + a^3 - \cancel{ab}$$

$$= a^3 + b^3$$

$$= R.H.S.$$

vec) L.H.S.

$$= \begin{vmatrix} 2\cos\phi & 1 & -\sin\phi \\ 0 & 1 & 0 \\ 2\sin\phi & 0 & \cos\phi \end{vmatrix}$$

$$= 2\cos\phi \begin{vmatrix} 1 & 0 \\ 0 & \cos\phi \end{vmatrix} - 1 \begin{vmatrix} 0 & 0 \\ 2\sin\phi & \cos\phi \end{vmatrix} + (-\sin\phi) \begin{vmatrix} 0 & 1 \\ 2\sin\phi & 0 \end{vmatrix}$$

$$= 2\cos\phi [(1)(\cos\phi) - (0)(0)] - 1 [(0)(\cos\phi) - (0)(2\sin\phi)] - \sin\phi [(0)(0) - (1)(2\sin\phi)]$$

$$= 2\cos\phi [\cos\phi - 0] - 1 [0 - 0] - \sin\phi [0 - 2\sin\phi]$$

$$= 2\cos\phi [\cos\phi] - 1 [0] - \sin\phi [-2\sin\phi]$$

$$= 2\cos^2\phi + 2\sin^2\phi$$

$$= 2(\cos^2\phi + \sin^2\phi)$$

$$= 2(1)$$

$$\because \cos^2\phi + \sin^2\phi = 1$$

$$= 2 = R.H.S.$$

Q. 8. (iii) L.H.S.

$$= \begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix}$$

$$= a \begin{vmatrix} c+a & b+c \\ a+b & c+a \end{vmatrix} - (b+c) \begin{vmatrix} b & b+c \\ c & c+a \end{vmatrix} + (a+b) \begin{vmatrix} b & c+a \\ c & a+b \end{vmatrix}$$

$$= a[(c+a)^2 - (a+b)(b+c)] - (b+c)[b(c+a) - c(b+c)] + (a+b)[b(a+b) - c(c+a)]$$

$$= a[c^2 + a^2 + 2ca - ab - ca - b^2 - bc] - (b+c)[bc + ab - bc - c^2] + (a+b)[ab + b^2 - c^2 - ca]$$

$$= a[a^2 - b^2 + c^2 - ab - bc + ca] - (b+c)[ab - c^2] + (a+b)[ab + b^2 - c^2 - ca]$$

$$= a^3 - ab^2 + ca^2 - a^2b - abc + ca^2 - ab^2 + bc^2 - abc + c^3 + a^2b + ab^2 - ca^2 - ca^2 + ab^2 + b^3 - bc^2 - abc$$

$$= a^3 + b^3 + c^3 - 3abc$$

$$= R.H.S.$$

C.H.S

$$\text{ix)} \begin{vmatrix} a+\lambda & b & c \\ a & b+\lambda & c \\ a & b & c+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c+\lambda & b & c \\ a+b+c+\lambda & b+\lambda & c \\ a+b+c+\lambda & b & c+\lambda \end{vmatrix} \quad C_1 + (C_2 + C_3)$$

$$(a+b+c+\lambda) \begin{vmatrix} 1 & b & c \\ 1 & b+\lambda & c \\ 1 & b & c+\lambda \end{vmatrix} \quad \begin{array}{l} \text{Taking } (a+b+c+\lambda) \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= (a+b+c+\lambda) \begin{vmatrix} 1 & b & c \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} \quad \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array}$$

$$= (a+b+c+\lambda) \begin{bmatrix} 1 & \lambda & 0 \\ 0 & \lambda \end{bmatrix} \quad \begin{array}{l} \text{Expanding} \\ \text{by } C_1. \end{array}$$

$$= (a+b+c+\lambda) [1(\lambda^2 - 0)]$$

$$= (a+b+c+\lambda)\lambda^2$$

$$= \text{R.H.S.}$$

x) C.H.S.

$$= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix} \begin{matrix} C_1 - C_2 \\ C_2 - C_3 \end{matrix}$$

Expanding by R_1

$$= 1 \begin{vmatrix} a-b & b-c \\ a^2-b^2 & b^2-c^2 \end{vmatrix}$$

$$= (a-b)(b^2-c^2) - (b-c)(a^2-b^2)$$

$$= (a-b)(b-c)(b+c) - (b-c)(a-b)(a+b)$$

$$= (a-b)(b-c) [b+c - (a+b)]$$

$$= (a-b)(b-c) (\cancel{b} + c - a - \cancel{b})$$

$$= (a-b)(b-c)(c-a)$$

$$= R.H.S$$

xi) C.H.S.

$$= \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a & a^2 \\ a+b+c & b & b^2 \\ a+b+c & c & c^2 \end{vmatrix} \begin{matrix} C_1 + C_2 \\ \\ \end{matrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} \text{taking} \\ (a+b+c) \text{ as} \\ \text{common from } C_1 \end{matrix}$$

$$= (a+b+c) \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} \begin{matrix} R_1 - R_3 \\ R_2 - R_3 \\ \end{matrix}$$

Expanding by C_1

$$= (a+b+c)(1) \begin{vmatrix} a-b & a^2-b^2 \\ b-c & b^2-c^2 \end{vmatrix}$$

$$= (a+b+c) [(a-b)(b^2-c^2) - (a^2-b^2)(b-c)]$$

$$= (a+b+c) [(a-b)(b-c)(b+c) - (a-b)(a+b)(b-c)]$$

$$= (a+b+c)(a-b)(b-c)[\cancel{b+c} - a - \cancel{b}]$$

$$= (a+b+c)(a-b)(b-c)(c-a)$$

$$= \text{R.H.S.}$$

Q4-

i) A_{12}

$$= (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ -2 & 1 \end{vmatrix} = (-1)^3 (0+0) \\ = (-1)(0) = 0$$

A_{22}

$$= (-1)^{2+2} \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} = (-1)^4 (1-6) \\ = (1)(-5) = -5$$

A_{32}

$$= (-1)^{3+2} \begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix} = (-1)^5 (0-0) \\ = (-1)(0) = 0$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 0 & 0 \\ -2 & 1 \end{vmatrix} + (-3) \begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix}$$

$$\begin{aligned}
 &= 1(-2-0) - 2(0-0) - 3(0-4) \\
 &= 1(-2) - 2(0) - 3(-4) \\
 &= -2 - 0 + 12 \\
 &= 10
 \end{aligned}$$

Q5:- Without Expansion

i) L.H.S.

$$= \begin{vmatrix} \alpha & \beta + \gamma & 1 \\ \beta & \gamma + \alpha & 1 \\ \gamma & \alpha + \beta & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \alpha + \beta + \gamma & \beta + \gamma & 1 \\ \alpha + \beta + \gamma & \gamma + \alpha & 1 \\ \alpha + \beta + \gamma & \alpha + \beta & 1 \end{vmatrix} \quad C_1 + C_2$$

$$= (\alpha + \beta + \gamma) \begin{vmatrix} 1 & \beta + \gamma & 1 \\ 1 & \gamma + \alpha & 1 \\ 1 & \alpha + \beta & 1 \end{vmatrix} \quad \begin{array}{l} \text{Taking } (\alpha + \beta + \gamma) \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= (\alpha + \beta + \gamma)(0) \quad \therefore C_1 = C_3$$

$$= 0$$

= R.H.S.

ii) L.H.S.

$$= \begin{vmatrix} 1 & 2 & 3x \\ 2 & 3 & 6x \\ 3 & 5 & 9x \end{vmatrix}$$

$$= 3x \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 5 & 3 \end{vmatrix} \begin{array}{l} \text{Taking '3x'} \\ \text{as common} \\ \text{from } C_3 \end{array}$$

$$= 3x (0) \quad \therefore C_1 = C_3$$

$$= 0$$

$$= \text{R.H.S.}$$

iii) L.H.S.

$$= \begin{vmatrix} 1 & a^2 & \frac{a}{bc} \\ 1 & b^2 & \frac{b}{ca} \\ 1 & c^2 & \frac{c}{ab} \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & a^2 & \frac{a^2bc}{bc} \\ 1 & b^2 & \frac{ab^2c}{ca} \\ 1 & c^2 & \frac{abc^2}{ab} \end{vmatrix} \begin{array}{l} \text{Multiply } C_3 \\ \text{by } abc \end{array}$$

$$= \frac{1}{abc} \begin{vmatrix} 1 & a^2 & a^2 \\ 1 & b^2 & b^2 \\ 1 & c^2 & c^2 \end{vmatrix}$$

$$= \frac{1}{abc} \times 0 \quad \because C_2 = C_3$$

$$= 0$$

$$= \text{R.H.S.}$$

$$\text{iv) L.H.S.} = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$= \begin{vmatrix} a-c & b-c & c-a \\ b-a & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix} \quad C_1 + C_2$$

$$= \begin{vmatrix} -(c-a) & b-c & c-a \\ -(a-b) & c-a & a-b \\ -(b-c) & a-b & b-c \end{vmatrix}$$

$$= -1 \cdot \begin{vmatrix} c-a & b-c & c-a \\ a-b & c-a & a-b \\ b-c & a-b & b-c \end{vmatrix} \quad \begin{array}{l} \text{Taking } (-1) \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= -1 \times 0 \quad \therefore C_1 = C_3$$

$$= 0$$

= R.H.S.

v) L.H.S.

$$= \begin{vmatrix} bc & ca & ab \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} bc & ca & ab \\ \frac{abc}{a} & \frac{abc}{b} & \frac{abc}{c} \\ a & b & c \end{vmatrix} \begin{array}{l} \text{Multiply } R_1 \\ \text{by } abc \end{array}$$

$$= \frac{1}{abc} \begin{vmatrix} bc & ca & ab \\ bc & ca & ab \\ a & b & c \end{vmatrix}$$

$$= \frac{1}{abc} \times 0 \quad \because R_1 = R_2$$

$$= 0$$

$$= R.H.S.$$

$$vi) \text{ L.H.S. } \begin{vmatrix} mn & l & l^2 \\ nl & m & m^2 \\ lm & n & n^2 \end{vmatrix}$$

$$= \frac{1}{lmn} \begin{vmatrix} lmn & l^2 & l^3 \\ lmn & m^2 & m^3 \\ lmn & n^2 & n^3 \end{vmatrix} \begin{array}{l} \text{Multiply } R_1 \\ \text{by } l, R_2 \text{ by} \\ \text{by } m \text{ \& } R_3 \text{ by } n \end{array}$$

$$= \frac{lmn}{lmn} \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix} \begin{array}{l} \text{Taking } lmn \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= \begin{vmatrix} 1 & l^2 & l^3 \\ 1 & m^2 & m^3 \\ 1 & n^2 & n^3 \end{vmatrix}$$

$$= \text{R.H.S.}$$

via) L.H.S.

$$= \begin{vmatrix} 2a & 2b & 2c \\ a+b & 2b & b+c \\ a+c & b+c & 2c \end{vmatrix}$$

$$= 2 \begin{vmatrix} a & b & c \\ a+b & 2b & b+c \\ a+c & b+c & 2c \end{vmatrix} \begin{array}{l} \text{Taking '2'} \\ \text{as common} \\ \text{from } R_1 \end{array}$$

$$= 2 \begin{vmatrix} a-b & b & c \\ a-b & 2b & b+c \\ a-b & b+c & 2c \end{vmatrix} \begin{array}{l} C_1 - C_2 \end{array}$$

$$= 2 \begin{vmatrix} a-b & b-c & c \\ a-b & b-c & b+c \\ a-b & b-c & 2c \end{vmatrix} \begin{array}{l} C_2 - C_3 \end{array}$$

$$= 2(a-b)(b-c) \begin{vmatrix} 1 & 1 & c \\ 1 & 1 & b+c \\ 1 & 1 & 2c \end{vmatrix} \begin{array}{l} \text{Taking } (a-b) \text{ as} \\ \text{common from } C_1 \\ \text{and } (b-c) \text{ from} \end{array}$$

$$= 2(a-b)(b-c) \times 0 \quad \because C_1 = C_2 \quad \underline{C_2}$$

$$= 0 = R.H.S.$$

iii) K.H.S.

$$= \begin{vmatrix} 7 & 2 & 7 \\ 6 & 3 & 5 \\ -3 & 5 & -3 \end{vmatrix} + \begin{vmatrix} 7 & 2 & -1 \\ 6 & 3 & -3 \\ -3 & 5 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 2 & 7-1 \\ 6 & 3 & 5-3 \\ -3 & 5 & -3+4 \end{vmatrix}$$

$$= \begin{vmatrix} 7 & 2 & 6 \\ 6 & 3 & 2 \\ -3 & 5 & 1 \end{vmatrix}$$

L.H.S.

ix) L.H.S.

$$= \begin{vmatrix} -a & 0 & c \\ 0 & a & -b \\ b & -c & 0 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} -ca & 0 & ca \\ 0 & ab & -ab \\ bc & -bc & 0 \end{vmatrix} \begin{array}{l} \text{Multiply } C_1 \text{ by 'c',} \\ C_2 \text{ by 'b' and} \\ C_3 \text{ by 'a'} \end{array}$$

$$= \frac{1}{abc} \begin{vmatrix} -ca & ca & ca \\ ab & 0 & -ab \\ 0 & -bc & 0 \end{vmatrix} \begin{array}{l} C_1 + C_2 \\ C_2 + C_3 \end{array}$$

$$= \frac{-1}{abc} \begin{vmatrix} ca & ca & ca \\ -ab & 0 & -ab \\ 0 & -bc & 0 \end{vmatrix} \begin{array}{l} \text{Taking } (-1) \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= \frac{-1}{abc} \times 0 \quad \therefore C_1 = C_3$$

$$= 0 = R.H.S$$

Q6 Find 'x'

$$\text{i) } \begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$$

$$\begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & x \end{vmatrix} - \begin{vmatrix} -1 & 4 \\ x & 0 \end{vmatrix} + \begin{vmatrix} -1 & 3 \\ x & 1 \end{vmatrix} = -30$$

$$3(0-4) - 1(0-4x) + x(-1-3x) = -30$$

$$-12 + 4x - x - 3x^2 + 30 = 0$$

$$-3x^2 + 3x + 18 = 0$$

$$-3(x^2 - x - 6) = 0$$

$$x^2 - x - 6 = \frac{0}{-3}$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x-3)(x+2) = 0$$

$$x-3=0, \quad x+2=0$$

$$x=3, \quad x=-2$$

$$\therefore \text{Sol. Set} = \{3, -2\}$$

$$\begin{array}{c|ccc|c} \infty) & 1 & x-1 & 3 & \\ & -1 & x+1 & 2 & = 0 \\ & 2 & -2 & x & \end{array}$$

$$1 \begin{vmatrix} x+1 & 2 \\ -2 & x \end{vmatrix} - (x-1) \begin{vmatrix} -1 & 2 \\ 2 & x \end{vmatrix} + 3 \begin{vmatrix} -1 & x+1 \\ 2 & -2 \end{vmatrix} = 0$$

$$1(x(x+1) - 2(-2)) - (x-1)(-x-4) + 3(2 - 2(x+1)) = 0$$

$$1(x^2 + x + 4) - (x-1)(x+4) + 3(2 - 2x - 2) = 0$$

$$x^2 + x + 4 + x^2 + 4x - x - 4 - 6x = 0$$

$$2x^2 - 2x = 0$$

$$2x(x-1) = 0$$

$$2x = 0, \quad x-1 = 0$$

$$x = \frac{0}{2}, \quad x = 1$$

$$x = 0$$

$$S. Set = \{0, 1\}$$

$$\begin{array}{c|ccc|c} \text{iii)} & 1 & 2 & 1 & \\ & 2 & x & 2 & = 0 \\ & 3 & 6 & x & \end{array}$$

$$1 \begin{vmatrix} x & 2 \\ 6 & x \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 3 & x \end{vmatrix} + 1 \begin{vmatrix} 2 & x \\ 3 & 6 \end{vmatrix} = 0$$

$$1(x^2 - 12) - 2(2x - 6) + 1(12 - 3x) = 0$$

$$x^2 - \cancel{12} - 4x + \cancel{12} + 12 - 3x = 0$$

$$x^2 - 7x + 12 = 0$$

$$x^2 - 4x - 3x - 12 = 0$$

$$x(x - 4) - 3(x - 4) = 0$$

$$(x - 4)(x - 3) = 0$$

$$x - 4 = 0, \quad x - 3 = 0$$

$$x = 4, \quad x = 3$$

$$\text{S. Set} = \{4, 3\}$$

$$\underline{Q7-} \quad \begin{array}{c} i) \end{array} \quad \left| \begin{array}{cccc} 3 & 4 & 2 & 7 \\ 2 & 5 & 0 & 3 \\ 1 & 2 & -3 & 5 \\ 4 & 1 & -2 & 6 \end{array} \right|$$

$$= 3 \left| \begin{array}{ccc|c} 5 & 0 & 3 & \\ 2 & -3 & 5 & -4 \\ 1 & -2 & 6 & \end{array} \right| \quad \left| \begin{array}{ccc|c} 2 & 0 & 3 & \\ 1 & -3 & 5 & \\ 4 & -2 & 6 & \end{array} \right|$$

$$+ 2 \left| \begin{array}{ccc|c} 2 & 5 & 3 & \\ 1 & 2 & 5 & -7 \\ 4 & 1 & 6 & \end{array} \right| \quad \left| \begin{array}{ccc|c} 2 & 5 & 0 & \\ 1 & 2 & -3 & \\ 4 & 1 & -2 & \end{array} \right|$$

$$= 3 [5(-18+10) - 0(12-5) + 3(-4+3)]$$

$$- 4 [2(-18+10) - 0(6-20) + 3(-2+12)]$$

$$+ 2 [2(12-5) - 5(6-20) + 3(1-8)]$$

$$- 7 [2(-4+3) - 5(-2+12) + 0(1-8)]$$

$$= 3 [5(-8) - 0(7) + 3(-1)] - 4 [2(-8) - 0(-14) + 3(10)]$$

$$+ 2 [2(7) - 5(-14) + 3(-7)] - 7 [2(-1) - 5(10) + 0(-7)]$$

$$\begin{aligned}
 &= 3[-40-0-3] - 4[-16+0+30] \\
 &+ 2[14+70-21] - 7[-2-50-0] \\
 &= 3[-43] - 4[14] + 2[63] - 7[-52] \\
 &= -129 - 56 + 126 + 364 \\
 &= 305
 \end{aligned}$$

$$\begin{array}{c|cccc}
 \text{11)} & 2 & 3 & 1 & -1 \\
 & 4 & 0 & 2 & 1 \\
 & 5 & 2 & -1 & 6 \\
 & 3 & -7 & 2 & -2
 \end{array}$$

$$\begin{array}{c|cccc|l}
 & 0 & 0 & 1 & 0 & C_1 - 2C_3 \\
 = & 0 & -6 & 2 & 3 & C_2 - 3C_3 \\
 & 7 & 5 & -1 & 5 & C_4 + C_3 \\
 & -1 & -13 & 2 & 0 &
 \end{array}$$

Expanding by R_1

$$\begin{array}{c|ccc}
 & 0 & -6 & 3 \\
 = 1(-1) & 7 & 5 & 5 \\
 & -1 & -13 & 0
 \end{array}$$

$$= \begin{vmatrix} 0 & 0 & 3 \\ 7 & 15 & 5 \\ -1 & -13 & 0 \end{vmatrix} \quad C_2 + 2C_3$$

Expanding by R_1

$$= 3 \begin{vmatrix} 7 & 15 \\ -1 & -13 \end{vmatrix}$$

$$= 3(-91 + 15)$$

$$= 3(-76) = -288$$

$$\begin{array}{l} \text{row} \\ \text{col} \end{array} \begin{vmatrix} -3 & 9 & 1 & 1 \\ 0 & 3 & -1 & 2 \\ 9 & 7 & -1 & 1 \\ -2 & 0 & 1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 & 1 \\ 6 & -15 & -3 & 2 \\ 12 & -2 & -2 & 1 \\ -5 & 9 & 9 & -1 \end{vmatrix} \begin{array}{l} C_1 + 3C_4 \\ C_2 - 9C_4 \\ C_3 - C_4 \end{array}$$

Expanding by R_1

$$= 1(-1)^{1+4} \begin{vmatrix} 6 & -15 & -3 \\ 12 & -2 & -2 \\ -5 & 9 & 2 \end{vmatrix}$$

$$= 1(-1)^5 3 \times 2 \begin{vmatrix} 2 & -5 & -1 \\ 6 & -1 & -1 \\ -5 & 9 & 2 \end{vmatrix} \begin{array}{l} \text{Taking 3 as common} \\ \text{from } R_1 \text{ and } R_2' \\ \text{from } R_2 \end{array}$$

$$= (-1)(6) \begin{vmatrix} 0 & 0 & -1 \\ 4 & 4 & -1 \\ -1 & -1 & 0 \end{vmatrix} \begin{array}{l} C_1 + 2C_3 \\ C_2 - 5C_3 \end{array}$$

Expanding by R_1

$$= -6(-1)^{1+3} \begin{vmatrix} 4 & 4 \\ -1 & -1 \end{vmatrix}$$

$$= -6(-1)^4 (-4 + 4)$$

$$= -6(1)(0)$$

$$= 0$$

Q.81- L.H.S.

$$= \begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix}$$

$$= \begin{vmatrix} x+3 & 1 & 1 & 1 \\ x+3 & x & 1 & 1 \\ x+3 & 1 & x & 1 \\ x+3 & 1 & 1 & x \end{vmatrix} \quad C_1 + (C_2 + C_3 + C_4)$$

$$= (x+3) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} \quad \begin{array}{l} \text{Taking } (x+3) \\ \text{as common} \\ \text{from } C_1 \end{array}$$

$$= (x+3) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & x-1 & 0 & 0 \\ 0 & 0 & x-1 & 0 \\ 0 & 0 & 0 & x-1 \end{vmatrix} \quad \begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{array}$$

	1	1	1	1	Partial (x-1)
$-(x+3)(x-1)(x-1)(x-1)$	0	1	0	0	as common
	0	0	1	0	from R_2
	0	0	0	1	R_3 & R_4

Expanding by C_1

$$= (x+3)(x-1)^3 (1)$$

$$= (x+3)(x-1)^3$$

$$= R.H.S.$$

	1	0	0	For Info
R_1	0	1	0	
	0	0	1	

$$= 1 [1(1-0)]$$

$$= 1 [1(1)]$$

$$= 1$$

Q9. ii)

$A =$

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$$

$A^t =$

$$\begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 9+16 & 6+4 & 3+4 & 6+12 \\ 6+4 & 4+1 & 2+1 & 4+3 \\ 3+4 & 2+1 & 1+1 & 2+3 \\ 6+12 & 4+3 & 2+3 & 4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 10 & 7 & 18 \\ 10 & 5 & 3 & 7 \\ 7 & 3 & 2 & 5 \\ 18 & 7 & 5 & 13 \end{bmatrix}$$

$$|AA^t| = \begin{vmatrix} 25 & 10 & 7 & 18 \\ 10 & 5 & 3 & 7 \\ 7 & 3 & 2 & 5 \\ 18 & 7 & 5 & 13 \end{vmatrix}$$

$$= 25 \begin{vmatrix} 5 & 3 & 7 \\ 3 & 2 & 5 \\ 7 & 5 & 13 \end{vmatrix} - 10 \begin{vmatrix} 10 & 3 & 7 \\ 7 & 2 & 5 \\ 18 & 5 & 13 \end{vmatrix}$$

$$+ 7 \begin{vmatrix} 10 & 5 & 7 \\ 7 & 3 & 5 \\ 18 & 7 & 13 \end{vmatrix} - 18 \begin{vmatrix} 10 & 5 & 3 \\ 7 & 3 & 2 \\ 18 & 7 & 5 \end{vmatrix}$$

$$= 25[5(1) - 3(4) + 7(1)] - 10[10(1) - 3(1) + 7(-1)]$$

$$+ 7[10(4) - 5(1) + 7(-5)] - 18[10(1) - 5(-1) + 3(5)]$$

$$= 25[5 - 12 + 7] - 10[10 - 3 - 7] + 7[40 - 5 - 35] - 18[10 + 5 - 15]$$

$$= 25[0] - 10[0] + 7[0] - 18[0]$$

$$= 0 - 0 - 0 - 0$$

$$= 0$$

Q10. - Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$|kA| = k^3 |A|$$

C.H.S.

$$= |kA|$$

$$= \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{vmatrix}$$

$$= k \cdot k \cdot k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{Taking 'k' as common from } R_1, R_2 \text{ \& } R_3$$

$$= k^3 |A|$$

$$= R.H.S.$$

Q11:-

$A =$

$$\begin{bmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{bmatrix}$$

As given,

$$|A| = 0$$

$$\begin{vmatrix} 4 & \lambda & 3 \\ 7 & 3 & 6 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$\begin{array}{c|c|c|c|c|c|c|c|c} 4 & 3 & 6 & -\lambda & 7 & 6 & +3 & 7 & 3 \\ \hline & 3 & 1 & & 2 & 1 & & 2 & 3 \end{array} = 0$$

$$4(3-18) - \lambda(7-12) + 3(21-6) = 0$$

$$4(-15) - \lambda(-5) + 3(15) = 0$$

$$-60 + 5\lambda + 45 = 0$$

$$5\lambda - 15 = 0$$

$$\lambda = \frac{15}{5}$$

$$\lambda = 3$$

Q12 ii)
$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

$$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 0 & -3 \\ -3 & 5 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 & -3 \\ 2 & 5 & 2 \end{vmatrix} + (-1) \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$$

$$= 2(5+0) - 3(5-0) - 1(-3-2)$$

$$= 2(5) - 3(5) - 1(-5)$$

$$= 10 - 15 + 5$$

$$= 10 - 10$$

$$= 0$$

It is singular.

Q13-

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

L.H.S.

$$= A^{-1}A$$

$$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} - 1 \begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix} + 0 \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$$

$$= 2(5+0) - 1(5-0) + 0(-3-2)$$

$$= 2(5) - 1(5) + 0$$

$$= 10 - 5$$

$$= 5 \neq 0$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} = (-1)^2 (5+0)$$
$$= (1)(5) = 5$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix} = (-1)^3 (5-0)$$
$$= (-1)(5) = -5$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = (-1)^4(-3-2) \\ = (1)(-5) = -5$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} = (-1)^3(5+0) \\ = (-1)(5) = -5$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 2 & 5 \end{vmatrix} = (-1)^4(10-0) \\ = (1)(10) = 10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} = (-1)^5(-6-2) \\ = (-1)(-8) = 8$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = (-1)^4(0-0) \\ = (1)(0) = 0$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = (-1)^5(0-0) \\ = (-1)(0) = 0$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (-1)^6(2-1) \\ = (1)(1) = 1$$

Write in transpose form

$$\text{Adj} A = \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix}$$

$$A^{-1}A = \frac{1}{5} \begin{bmatrix} 5 & -5 & 0 \\ -5 & 10 & 0 \\ -5 & 8 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 10-5+0 & 5-5+0 & 0+0+0 \\ -10+10+0 & -5+10+0 & 0+0+0 \\ -10+8+2 & -5+8-3 & 0+0+5 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

$$= R.H.S.$$

Q14:-

$$i) A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

C.H.S.

$$= (AB)^{-1}$$

$$AB = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+8 & 1-2 \\ 3+0 & -1-0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 \\ 3 & -1 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 5 & -1 \\ 3 & -1 \end{vmatrix}$$

$$= (5)(-1) - (-1)(3)$$

$$= -5 + 3$$

$$= -2 \neq 0$$

$$\text{Adj}(AB) = \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \text{Adj} AB$$

$$= \frac{1}{-2} \begin{bmatrix} -1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{5}{2} \end{bmatrix}$$

R.H.S.

$$= B^{-1}A^{-1}$$

$$|B| = \begin{vmatrix} -3 & 1 \\ 4 & -1 \end{vmatrix}$$

$$= (-3)(-1) - (1)(4)$$

$$= 3 - 4$$

$$= -1 \neq 0$$

$$\text{Adj}^{\circ} B = \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj}^{\circ} B$$

$$= \frac{1}{-1} \begin{bmatrix} -1 & -1 \\ -4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix}$$

$$= (1)(0) - (2)(-1)$$

$$= 0 + 2$$

$$= 2 \neq 0$$

$$\text{Adj}^{\circ} A = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}^{\circ} A$$

$$= \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 0+1 & -2+1 \\ 0+3 & -8+3 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{5}{2} \end{bmatrix}$$

Hence,

$$L.H.S. = R.H.S.$$

Q15:- $(AB)^t = B^t A^t$

L.H.S. $= (AB)^t$

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1-3+0 & 1-2-2 \\ 0+9+0 & 0+6-1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -2 & -3 \\ 9 & 5 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix}$$

$$R.H.S. = B^t A^t$$

$$B^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}, A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 1-3+0 & 0+9+0 \\ 1-2-2 & 0+6-1 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} -2 & -3 \\ 9 & 5 \end{bmatrix}$$

$$L.H.S. = R.H.S.$$

Q17.

i) $(AB)^{-1} = B^{-1}A^{-1}$
L.H.S.

$$= (AB)^{-1}$$

$$= (AB I)^{-1}$$

$$= (A B A A^{-1})^{-1} \quad \because I = A A^{-1}$$

$$= A^{-1} B^{-1} A^{-1} (A^{-1})^{-1}$$

$$= A^{-1} B^{-1} A^{-1} A \quad \because (A^{-1})^{-1} = A$$

$$= A^{-1} A B^{-1} A^{-1}$$

$$= I B^{-1} A^{-1} \quad \because A^{-1} A = I$$

$$= B^{-1} A^{-1}$$

$$= R.H.S.$$

ii) $(A^{-1})^{-1} = A$

L.H.S.

$$= (A^{-1})^{-1}$$

$$= (A^{-1} I)^{-1}$$

$$= (A^{-1} A A^{-1})^{-1} \quad \because I = A A^{-1}$$

$$= (A^{-1})^{-1} A^{-1} (A^{-1})^{-1}$$

$$= A A^{-1} A$$

$$= I A$$

$$= A = R.H.S.$$

$$\because (A^{-1})^{-1} = A$$

$$\because A A^{-1} = I$$