Exercise 3.4

Q.1:- $A=\begin{bmatrix}1 & -2 & 5\\ -2 & 3 & -1\\ 5 & -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}-3 & 1 & -2\\ 1 & 0 & -1\\ -2 & -1 & 2\end{bmatrix}$

$$A+B=\begin{bmatrix}1 & -2 & 5\\ -2 & 3 & -1\\ 5 & -1 & 0\end{bmatrix}+\begin{bmatrix}-3 & 1 & -2\\ 1 & 0 & -1\\ -2 & -1 & 2\end{bmatrix}$$

$$A+B=\begin{bmatrix}-2 & -1 & 3\\ -1 & 3 & -2\\ 3 & -2 & 2\end{bmatrix}$$

$$(A+B)^t=\begin{bmatrix}-2 & -1 & 3\\ -1 & 3 & -2\\ 3 & -2 & 2\end{bmatrix}$$

$(A+B)^t = A+B$

Hence, $A+B$ is symmetric.

Q2.-

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$$

i) $A + A^t$ is symmetric

$$A^t = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A + A^t = \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

$$(A + A^t)^t = \begin{bmatrix} 2 & 5 & -1 \\ 5 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

$(A+A^t)^t = A+A^t$

Hence, $A+A^t$ is symmetric.

ii) $A-A^t$ is skew symmetric

$$A-A^t = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 2 & -1 \\ -1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A-A^t = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$$(A-A^t)^t = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 4 \\ 1 & -4 & 0 \end{bmatrix}$$

$$(A-A^t)^t = -\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -4 \\ -1 & 4 & 0 \end{bmatrix}$$

$(A-A^t)^t = -(A-A^t)$

Hence, $A-A^t$ is skew-symmetric.

(3)

i) If A is square matrix of order 3, show that $A+A^t$ is symmetric.

$$(A+A^t)^t = A^t + (A^t)^t$$

$$= A^t + A \quad \because (A^t)^t = A$$

$$= A + A^t$$

Hence, $A+A^t$ is symmetric

ii) $A-A^t$ is skew-symmetric

$$(A-A^t)^t = A^t - (A^t)^t$$

$$= A^t - A \quad \because (A^t)^t = A$$

$$= -A + A^t$$

$$= -(A-A^t)$$

Hence, $A-A^t$ is skew-symmetric

Q4.-

As $A^t = A$ and $B^t = B$

$(AB)^t = AB$

L.H.S.

$= (AB)^t$

$= B^t A^t$

$= BA \qquad \because B^t = B,\ A^t = A$

$= AB$

$=$ R.H.S

Q5.- Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$A^t = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

$$AA^t = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{bmatrix}$$

$$AA^t = \begin{bmatrix} a_{11}^2+a_{12}^2+a_{13}^2 & a_{11}a_{21}+a_{12}a_{22}+a_{13}a_{23} \\ a_{21}a_{11}+a_{22}a_{12}+a_{23}a_{13} & a_{21}^2+a_{22}^2+a_{23}^2 \end{bmatrix}$$

$$(AA^t)^t = \begin{bmatrix} a_{11}^2+a_{12}^2+a_{13}^2 & a_{11}a_{21}+a_{12}a_{22}+a_{13}+a_{23} \\ a_{21}a_{11}+a_{22}a_{12}+a_{23}a_{13} & a_{21}^2+a_{22}^2+a_{23}^2 \end{bmatrix}$$

$$(AA^t)^t = AA^t$$

Hence, AA^t is symmetric.

(b) i) $A+(\bar{A})^t$ is hermitian

$$A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$$

$$\bar{A} - \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

$$A+(\bar{A})^t = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} + \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

$$A+(\bar{A})^t = \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix}$$

$$\overline{A+(\bar{A})^t} = \begin{bmatrix} 0 & 2-i \\ 2+i & 0 \end{bmatrix}$$

$$\left(\overline{A+(\bar{A})^t}\right)^t = \begin{bmatrix} 0 & 2+i \\ 2-i & 0 \end{bmatrix}$$

$$\left(\overline{A+(\bar{A})^t}\right)^t = A+(\bar{A})^t$$

Hence, $A+(\bar{A})^t$ is hermitian

Q6.-

ii) $A-(\bar{A})^t$ is skew-hermitian

$$A = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -i & 1-i \\ 1 & i \end{bmatrix}$$

$$(\bar{A})^t = \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

$$A-(\bar{A})^t = \begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix} - \begin{bmatrix} -i & 1 \\ 1-i & i \end{bmatrix}$$

$$= \begin{bmatrix} 2i & i \\ +i & -2i \end{bmatrix}$$

$$\overline{A-(\bar{A})^t} = \begin{bmatrix} -2i & -i \\ -i & +2i \end{bmatrix}$$

$$\left(\overline{A - (\bar{A})^t}\right)^t = \begin{bmatrix} -2i & -i \\ -i & 2i \end{bmatrix}$$

$$\left(\overline{A - (\bar{A})^t}\right)^t = - \begin{bmatrix} +2i & i \\ i & -2i \end{bmatrix}$$

$$\left(\overline{A - (\bar{A})^t}\right)^t = - \left(A - (\bar{A})^t\right)$$

Hence, $A - (\bar{A})^t$ is skew-hermitian

Q7. $A^t = A$ ✓

or $A^t = -A$

$= A^2$

$= AA$

$= (AA)^t$

$= A^t A^t$

$= AA \qquad \because A^t = A$

$= A^2$

Hence, A^2 is symmetric

Q8. $A = \begin{bmatrix} 1 \\ 1+i \\ 0 \\ i \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix}$

$(\bar{A})^t = \begin{bmatrix} 1 & 1-i & -i \end{bmatrix}$

$A(\bar{A})^t = \begin{bmatrix} 1 \\ 1-i \\ -i \end{bmatrix} \begin{bmatrix} 1 & 1+i & i \end{bmatrix}$

$= \begin{bmatrix} 1 & 1+i & i \\ 1-i & 1-i^2 & i-i^2 \\ -i & -i-i^2 & -i^2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1+i & i \\ 1-i & 1-(-1) & i-(-1) \\ -i & -i-(-1) & -(-1) \end{bmatrix}$

$$A(\bar{A})^t = \begin{bmatrix} 1 & 1+i & i \\ 1-i & 2 & 1+i \\ -i & 1-i & 1 \end{bmatrix}$$

Q8:- Using Row Operation

i) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

$$\underset{\sim}{R} = \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ -2 & -2 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 2 & -4 & 2 & 0 & 1 \end{array}\right] \quad R_3+2R_1 \quad \boxed{-2+2(1)}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & -3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 2 & -4 & 2 & 0 & 1 \end{array}\right] \quad R_2\left(\frac{1}{-2}\right)$$

$R_1 - 2R_2$

$-3-2(0)$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -4 & 2 & 1 & 1 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ R_3-2R_2 \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] R_3\left(\frac{1}{-4}\right)$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{4} & -3/4 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] R_1+3R_3$$

$-3+3(1)$
$=0$

$1+3\left(-\frac{1}{2}\right)$
$=\frac{2-3}{2}=-\frac{1}{2}$

$1+3\left(-\frac{1}{4}\right)$
$=\frac{4-3}{4}=\frac{1}{4}$

$0+3\left(-\frac{1}{4}\right)$

Hence,

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

Q9:-

ii) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

$$\underset{\sim}{R} = \left[\begin{array}{ccc:ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc:ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{array}\right] R_3 - R_1$$

$$= \left[\begin{array}{ccc:ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & -2 & 3 & -1 & 0 & 1 \end{array}\right] R_2(-1)$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & -3 & -1 & -2 & 1 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ R_3 + 2R_2 \\ 0+2(-1) \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1/3 & 2/3 & -1/3 \end{array}\right] R_3\left(-\frac{1}{3}\right)$$

$0+\frac{3}{3}=1$

$-1+\frac{6}{3}=-1+2$

$0-\frac{3}{3}=-1$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{array}\right] \begin{array}{l} R_1-5R_3 \\ R_2+3R_3 \end{array}$$

$1-\frac{5}{3}=\frac{3-5}{3}$

$2-\frac{10}{3}=\frac{6-10}{3}$

$0+\frac{5}{3}$

Hence,

$$= \begin{bmatrix} -\frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \\ 1 & 1 & -1 \\ \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

It is the inverse.

Q9:-

iii)
$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$0$$

$$\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}$$ دوسرے

$$\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$ تیسرے

$$\underset{\sim}{R} = \left[\begin{array}{ccc:ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc:ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 7 & -4 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] R_2 - 2R_1$$

$$= \left[\begin{array}{ccc:ccc} 1 & -3 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{7} & \frac{-2}{7} & \frac{1}{7} & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right] R_2\left(\frac{1}{7}\right)$$

$$2 + 3\left(\frac{-4}{7}\right) = \frac{14-12}{7}$$

$$1 + 3\left(\frac{-2}{7}\right) = \frac{7-6}{7}$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & \frac{2}{7} & \frac{1}{7} & \frac{3}{7} & 0 \\ 0 & 1 & -\frac{4}{7} & \frac{-2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & \frac{3}{7} & \frac{-2}{7} & \frac{1}{7} & 1 \end{array}\right] \begin{matrix} R_1 + 3R_2 \\ R_3 + R_2 \end{matrix}$$

$$1 - \frac{4}{7} = \frac{7-4}{7}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{7} & \frac{1}{7} & \frac{3}{7} & 0 \\ 0 & 1 & -\frac{4}{7} & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{7}{3} \end{array}\right] R_3\left(\frac{7}{3}\right)$$

$\frac{1}{7} - \frac{2}{7}\left(-\frac{2}{3}\right)$

$= \frac{1}{7} + \frac{4}{21} = \frac{3}{21} + \frac{4}{21} = \frac{\cancel{7}\,1}{\cancel{21}\,3}$

$\frac{3}{7} - \frac{2}{7}\left(\frac{1}{3}\right)$

$= \frac{3}{7} - \frac{2}{21} = \frac{9}{21} - \frac{2}{21}$

$\frac{7}{21} = \frac{1}{3}$

$0 - \frac{2}{\cancel{7}}\left(\frac{\cancel{7}}{3}\right)$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & -\frac{4}{7} & -\frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{7}{3} \end{array}\right] R_1 - \frac{2}{7} R_3$$

$\boxed{\begin{array}{l} \frac{2}{7} - \frac{2}{7}(1) \\ \frac{2}{7} - \frac{2}{7} = 0 \end{array}}$

$\frac{-2}{7} + \frac{4}{7}\left(\frac{-2}{3}\right)$

$= \frac{-2}{7} - \frac{8}{21} = \frac{-6}{21} - \frac{8}{21}$

$= \frac{-14}{21} = \frac{-2}{3}$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & -\frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ 0 & 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{7}{3} \end{array}\right] R_2 + \frac{4}{7} R_3$$

$\frac{1}{7} + \frac{4}{7}\left(\frac{1}{3}\right)$

$= \frac{1}{7} + \frac{4}{21} = \frac{3}{21} + \frac{4}{21}$

$= \frac{7}{21} = \frac{1}{3}$

$0 + \frac{4}{\cancel{7}}\left(\frac{\cancel{7}}{3}\right) = \frac{4}{3}$

Hence, Inverse is

$$= \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ -2/3 & 1/3 & 4/3 \\ -2/3 & 1/3 & 7/3 \end{bmatrix}$$

Q 9:- Inverse

i) Using Adjoint Method

Let

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{vmatrix}$$

$$|A| = 1\begin{vmatrix} -2 & 0 \\ -2 & 2 \end{vmatrix} - 2\begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix}$$

$|A| = 1(-4+0) - 2(0+0) - 3(0-4)$

$|A| = 1(-4) - 2(0) - 3(-4)$

$|A| = -4 - 0 + 12$

$|A| = 8 \neq 0$

A^{-1} exists

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 0 \\ -2 & 2 \end{vmatrix} = (-1)^2(-4+0) = (1)(-4) = -4$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} = (-1)^3(0-0) = 0$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & -2 \\ -2 & -2 \end{vmatrix} = (-1)^4(0-4) = (1)(-4) = -4$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -3 \\ -2 & 2 \end{vmatrix} = (-1)^3(4-6) = (-1)(-2) = 2$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -3 \\ -2 & 2 \end{vmatrix} = (-1)^4(2-6) = (1)(-4) = -4$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix} = (-1)^5(-2+4) = (-1)(2) = -2$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -3 \\ -2 & 0 \end{vmatrix} = (-1)^4(0-6) = (1)(-6) = -6$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -3 \\ 0 & 0 \end{vmatrix} = (-1)^5(0-0) = 0$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = (-1)^6(-2-0)$$

$$= 1(-2) = -2$$

$$\text{Adj} A = \begin{bmatrix} -4 & 0 & -4 \\ 2 & -4 & -2 \\ -6 & 0 & -2 \end{bmatrix}^t$$

$$\text{Adj} A = \begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj} A$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 2 & -6 \\ 0 & -4 & 0 \\ -4 & -2 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -4/8 & 2/8 & -6/8 \\ 0/8 & -4/8 & 0/8 \\ -4/8 & -2/8 & -2/8 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/2 & 1/4 & -3/4 \\ 0 & -1/2 & 0 \\ -1/2 & -1/4 & -1/4 \end{bmatrix}$$

Part (ii) & (iii) are same.

Q9:- Using Column Operations

i) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

$$\underset{\sim}{C} = \left[\begin{array}{ccc} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \\ \hdashline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & -2 & 0 \\ -2 & 2 & -4 \\ \hdashline 1 & -2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ C_2-2C_1 \\ C_3+3C_1 \\ \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & -4 \\ \hline 1 & 1 & 3 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{array}\right] \quad C_2\left(-\frac{1}{2}\right)$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & 1 \\ \hline 1 & 1 & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{4} \end{array}\right] \quad C_3\left(-\frac{1}{4}\right)$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline -\frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} C_1+2C_3 \\ C_2+C_3 \end{array}$$

Hence, inverse is

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{4} & -\frac{3}{4} \\ 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

ii) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$

$$\underset{\sim}{C} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 1 & 0 & 2 \\ \hdashline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & 3 \\ 1 & -2 & 3 \\ \hline 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} C_2-2C_1 \\ C_3+C_1 \\ \\ \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 1 & 2 & 3 \\ \hline 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ C_2(-1) \\ \\ \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & -3 \\ \hline 1 & 2 & -5 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ C_3-3C_2 \\ \\ \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \\ \hline 1 & 2 & 5/3 \\ 0 & -1 & -1 \\ 0 & 0 & -1/3 \end{array}\right] \quad C_3\left(-\frac{1}{3}\right)$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline -\frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \\ 1 & 1 & -1 \\ \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{array}\right] \quad \begin{array}{l} C_1 - C_3 \\ C_2 - 2C_3 \end{array}$$

Hence, the inverse is

$$= \begin{bmatrix} -\frac{2}{3} & -\frac{4}{3} & \frac{5}{3} \\ 1 & 1 & -1 \\ \frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

iii)

$$\begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\underset{\sim}{C} = \left[\begin{array}{ccc} 1 & -3 & 2 \\ 2 & 1 & 0 \\ 0 & -1 & 1 \\ \hdashline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 7 & -4 \\ 0 & -1 & 1 \\ \hdashline 1 & 3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ C_2+3C_1 \\ C_3-2C_1 \\ \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & -4 \\ 0 & -\frac{1}{7} & 1 \\ \hdashline 1 & \frac{3}{7} & -2 \\ 0 & \frac{1}{7} & 0 \\ 0 & 0 & 1 \end{array}\right] \quad C_2\left(\frac{1}{7}\right)$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{2}{7} & -\frac{1}{7} & \frac{3}{7} \\ \hdashline \frac{1}{7} & \frac{3}{7} & -\frac{2}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \\ 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} C_1 - 2C_2 \\ C_3 + 4C_2 \end{array}$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{2}{7} & -\frac{1}{7} & 1 \\ \hdashline \frac{1}{7} & \frac{3}{7} & -\frac{2}{3} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{3} \\ 0 & 0 & 7/3 \end{array}\right] \quad C_3\left(\frac{7}{3}\right)$$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hdashline \frac{1}{3} & \frac{1}{3} & \frac{-2}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{7}{3} \end{array}\right] \begin{array}{l} \\ C_1 - \left(\frac{2}{7}\right) C_3 \\ \\ C_2 + \left(\frac{1}{7}\right) C_3 \\ \\ \\ \end{array}$$

Hence, the inverse is

$$= \left[\begin{array}{ccc} \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{4}{3} \\ -\frac{2}{3} & \frac{1}{3} & \frac{7}{3} \end{array}\right]$$

Q10:-

i) $\begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & -6 & 5 & 1 \\ 3 & 5 & 4 & -3 \end{bmatrix}$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & -4 & 1 & -1 \\ 0 & 8 & -2 & -6 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \\ \ \end{matrix}$$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 8 & -2 & -6 \end{bmatrix} \begin{matrix} R_2\left(-\frac{1}{4}\right) \\ \ \\ \ \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{7}{4} & \frac{5}{4} \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & -8 \end{bmatrix} \begin{matrix} R_1 + R_2 \\ R_3 - 8R_2 \\ \ \end{matrix}$$

Rank = 3

Rank is the number of non-zero rows.

ii) $\begin{bmatrix} 1 & -4 & -7 \\ 2 & -5 & 1 \\ 1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & -4 & -7 \\ 0 & 3 & 15 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - R_1 \\ R_4 - 3R_1 \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & -4 & -7 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \\ 0 & 5 & 25 \end{bmatrix} \begin{matrix} R_2\left(\frac{1}{3}\right) \\ \\ \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 0 & 13 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1 - 4R_2 \\ R_3 - 2R_2 \\ R_4 - 5R_2 \\ \end{matrix}$$

Rank = 2

iii) $\begin{bmatrix} 3 & -1 & 3 & 0 & -1 \\ 1 & 2 & -1 & -3 & -2 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{bmatrix}$

$$\underset{\sim}{R} = \begin{bmatrix} 1 & 2 & -1 & -3 & -2 \\ 3 & -1 & 3 & 0 & -1 \\ 2 & 3 & 4 & 2 & 5 \\ 2 & 5 & -2 & -3 & 3 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$= \begin{bmatrix} 1 & 2 & -1 & -3 & -2 \\ 0 & -7 & 6 & 9 & 5 \\ 0 & -1 & 6 & 8 & 9 \\ 0 & 1 & 0 & 3 & 7 \end{bmatrix} \begin{matrix} R_2-3R_1 \\ R_3-2R_1 \\ R_4-2R_1 \\ \\ \end{matrix}$$

$$= \begin{bmatrix} 1 & 2 & -1 & -3 & -2 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & -1 & 6 & 8 & 9 \\ 0 & -7 & 6 & 9 & 5 \end{bmatrix} R_2 \leftrightarrow R_4$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 6 & 11 & 16 \\ 0 & 0 & 6 & 30 & 54 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ R_3+R_2 \\ R_4+7R_2 \\ \\ \end{array}$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 6 & 30 & 54 \\ 0 & 0 & 6 & 11 & 16 \end{array}\right] \quad R_3 \longleftrightarrow R_4$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1 & -9 & -16 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 6 & 11 & 16 \end{array}\right] \quad R_3\left(\frac{1}{6}\right)$$

$$= \left[\begin{array}{ccccc} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & -19 & -38 \end{array}\right] \begin{array}{l} R_1+R_3 \\ R_4-6R_3 \\ \\ \\ \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -4 & -7 \\ 0 & 1 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 & 9 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \left(-\frac{1}{19}\right)R_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{array}{l} R_1+4R_4 \\ R_2-3R_4 \\ R_3-5R_4 \\ \end{array}$$

Rank = 4