Ex#2.1

Derivative By Definition

Steps to Be Taken

① let the given value as y

② Add y with δy as $y+\delta y$

③ Subtracting ① & ② to get 'δy'

④ Divided by δy to get $\frac{\delta y}{\delta x}$

⑤ Taking $\lim_{\delta x \to 0}$ as

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

(2)

i) $2x^2+1$

Let

$$y = 2x^2+1 \quad \text{— (1)}$$

$$y+\delta y = 2(x+\delta x)^2+1 \quad \text{— (2)}$$

Subtracting (1) & (2).

$$\cancel{y}+\delta y-\cancel{y} = 2(x+\delta x)^2+1-(2x^2+1)$$

$$\delta y = 2(x^2+\delta x^2+2x\delta x)+\cancel{1}-2x^2-\cancel{1}$$

$$\delta y = \cancel{2x^2}+2\delta x^2+4x\delta x\cancel{-2x^2}$$

$$\delta y = 2\delta x^2+4x\delta x$$

$$\delta y = \delta x(2\delta x+4x)$$

$$\frac{\delta y}{\delta x} = 2\delta x+4x$$

Taking $\lim\limits_{\delta x\to 0}$

$$\boxed{\lim_{\delta x\to 0}\frac{\delta y}{\delta x}} = \lim_{\delta x\to 0}(2\delta x+4x)$$

Derivative

$$= 2(0)+4x$$

$$= 4x$$

ii) $2-\sqrt{x}$

let

$$y = 2-\sqrt{x} \quad \text{——} \quad ①$$

$$y+\delta y = 2-\sqrt{x+\delta x} \quad \text{——} \quad ②$$

Subtracting ① & ②

$$\cancel{y}+\delta y - \cancel{y} = 2-\sqrt{x+\delta x} - (2-\sqrt{x})$$

$$\delta y = \cancel{2}-\sqrt{x+\delta x} - \cancel{2} + \sqrt{x}$$

$$\delta y = \sqrt{x} - \sqrt{x+\delta x}$$

$$\delta y = \frac{\sqrt{x}-\sqrt{x+\delta x}}{1} \times \frac{\sqrt{x}+\sqrt{x+\delta x}}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\delta y = \frac{(\sqrt{x})^2 - (\sqrt{x+\delta x})^2}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\delta y = \frac{\cancel{x} - \cancel{x} - \delta x}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\delta y = \frac{-\delta x}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$\frac{\delta y}{\delta x} = \frac{-1}{\sqrt{x}+\sqrt{x+\delta x}}$$

Taking $\lim_{\delta x \to 0}$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{-1}{\sqrt{x}+\sqrt{x+\delta x}}$$

$$= \frac{-1}{\sqrt{x}+\sqrt{x+0}}$$

$$= \frac{-1}{\sqrt{x}+\sqrt{x}}$$

$$= \frac{-1}{2\sqrt{x}}$$

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iii) $\frac{1}{\sqrt{x}}$

Let

$$y = \frac{1}{\sqrt{x}} \quad \text{---} (1)$$

$$y+\delta y = \frac{1}{\sqrt{x+\delta x}} \quad \text{---} (2)$$

Subtracting (1) & (2)

$$y+\delta y - y = \frac{1}{\sqrt{x+\delta x}} - \frac{1}{\sqrt{x}}$$

$$\delta y = \frac{\sqrt{x} - \sqrt{x+\delta x}}{\sqrt{x+\delta x}\sqrt{x}}$$

$$\delta y = \frac{\sqrt{x} - \sqrt{x+\delta x}}{\sqrt{x+\delta x}\sqrt{x}} \times \frac{\sqrt{x} + \sqrt{x+\delta x}}{\sqrt{x} + \sqrt{x+\delta x}}$$

$$\delta y = \frac{(\sqrt{x})^2 - (\sqrt{x+\delta x})^2}{(\sqrt{x+\delta x}\sqrt{x})(\sqrt{x} + \sqrt{x+\delta x})}$$

$$\delta y = \frac{x - x - \delta x}{(\sqrt{x+\delta x}\sqrt{x})(\sqrt{x} + \sqrt{x+\delta x})}$$

$$\delta y = \frac{-\delta x}{(\sqrt{x+\delta x}\sqrt{x})(\sqrt{x} + \sqrt{x+\delta x})}$$

$$\frac{\delta y}{\delta x} = \frac{-1}{(\sqrt{x+\delta x}\sqrt{x})(\sqrt{x} + \sqrt{x+\delta x})}$$

Taking $\lim\limits_{\delta x \to 0}$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{-1}{(\sqrt{x+\delta x}\sqrt{x})(\sqrt{x} + \sqrt{x+\delta x})}$$

$$= \frac{-1}{(\sqrt{x+0}\sqrt{x})(\sqrt{x} + \sqrt{x+0})}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{-1}{(\sqrt{x}\sqrt{x})(\sqrt{x}+\sqrt{x})}$$

$$= \frac{-1}{(\sqrt{x})^2\, 2\sqrt{x}}$$

$$= \frac{-1}{2x\sqrt{x}}$$

ii) $\frac{1}{x^3}$

Let

$$y = \frac{1}{x^3} \quad \text{—— (1)}$$

$$y + \delta y = \frac{1}{(x+\delta x)^3} \quad \text{—— (2)}$$

Subtracting (1) & (2)

$$\cancel{y} + \delta y - \cancel{y} = \frac{1}{(x+\delta x)^3} - \frac{1}{x^3}$$

$$\delta y = \frac{x^3 - (x+\delta x)^3}{(x+\delta x)^3\, x^3}$$

$$\delta y = \frac{[x-(x+\delta x)][x^2 + x(x+\delta x) + (x+\delta x)^2]}{(x+\delta x)^3 x^3}$$

$$\delta y = \frac{(x - x - \delta x)\left[x^2 + x^2 + x\delta x + x^2 + \delta x^2 + 2x\delta x\right]}{(x+\delta x)^3 x^3}$$

$$\delta y = \frac{-\delta x\left[3x^2 + x\delta x + \delta x^2 + 2x\delta x\right]}{(x+\delta x)^3 x^3}$$

$$\frac{\delta y}{\delta x} = \frac{-\left[3x^2 + x\delta x + \delta x^2 + 2x\delta x\right]}{(x+\delta x)^3 x^3}$$

Taking $\lim\limits_{\delta x \to 0}$.

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{-\left[3x^2 + x\delta x + \delta x^2 + 2x\delta x\right]}{(x+\delta x)^3 x^3}$$

$$= \frac{-\left[3x^2 + x(0) + (0)^2 + 2x(0)\right]}{(x+0)^3 x^3}$$

$$= \frac{-[3x^2]}{x^3 x^3}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^{6-2}} = \frac{-3}{x^4}$$

v) $\frac{1}{x-a}$

let

$$y = \frac{1}{x-a} \quad \text{——— ①}$$

$$y+\delta y = \frac{1}{x+\delta x-a} \quad \text{——— ②}$$

Subtracting ① & ②

$$\cancel{y}+\delta y-\cancel{y} = \frac{1}{x+\delta x-a} - \frac{1}{x-a}$$

$$\delta y = \frac{x-a-(x+\delta x-a)}{(x+\delta x-a)(x-a)}$$

$$\delta y = \frac{\cancel{x}-\cancel{a}-\cancel{x}-\delta x+\cancel{a}}{(x+\delta x-a)(x-a)}$$

$$\delta y = \frac{-\delta x}{(x+\delta x-a)(x-a)}$$

$$\frac{\delta y}{\delta x} = \frac{-1}{(x+\delta x-a)(x-a)}$$

Taking $\lim_{\delta x \to 0}$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{-1}{(x+\delta x-a)(x-a)}$$

$$= \frac{-1}{(x+0-a)(x-a)}$$

$$= \frac{-1}{(x-a)(x-a)}$$

$$= \frac{-1}{(x-a)^2}$$

vi) $x(x-3)$

Let

$$y = x(x-3) \text{ ——— } ①$$

$$y+\delta y = (x+\delta x)(x+\delta x-3) \text{ ——— } ②$$

Subtracting ① & ②

$$\cancel{y}+\delta y-\cancel{y} = (x+\delta x)(x+\delta x-3) - x(x-3)$$

$$\delta y = \cancel{x^2} + x\delta x - \cancel{3x} + x\delta x + \delta x^2 - 3\delta x - \cancel{x^2} + \cancel{3x}$$

$$\delta y = 2x\delta x + \delta x^2 - 3\delta x$$

$$\delta y = \delta x(2x+\delta x-3)$$

$$\frac{\delta y}{\delta x} = 2x+\delta x-3$$

Taking $\lim\limits_{\delta x \to 0}$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} (2x+\delta x-3)$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 2x + 0 - 3$$
$$= 2x - 3$$

vii) $\dfrac{2}{x^4}$

Let

$$y = \frac{2}{x^4} \text{ ——— } (1)$$

$$y + \delta y = \frac{2}{(x+\delta x)^4} \text{ ——— } (2)$$

Subtracting (1) & (2)

$$\cancel{y} + \delta y - \cancel{y} = \frac{2}{(x+\delta x)^4} - \frac{2}{x^4}$$

$$\delta y = \frac{2x^4 - 2(x+\delta x)^4}{(x+\delta x)^4 x^4}$$

$$\delta y = \frac{2x^4 - 2(x+\delta x)^2 (x+\delta x)^2}{(x+\delta x)^4 x^4}$$

$$\delta y = \frac{2x^4 - 2(x^2+\delta x^2+2x\delta x)(x^2+\delta x^2+2x\delta x)}{(x+\delta x)^4 x^4}$$

$$\delta y = \frac{2x^4 - 2[x^4 + x^2\delta x^2 + 2x^3\delta x + x^2\delta x^2 + \delta x^4 + 2x\delta x^3 + 2x^3\delta x + 2x\delta x^3 + 4x^2\delta x^2]}{(x+\delta x)^4 x^4}$$

$$\delta y = \frac{2x^4 - 2(x^4 + 6x^2\delta x^2 + 4x^3\delta x + \delta x^4 + 4x\delta x^3)}{(x+\delta x)^4 x^4 \cancel{(x+\delta x)}}$$

$$\delta y = \frac{\cancel{2x^4} - \cancel{2x^4} - 12x^2\delta x^2 - 8x^3\delta x - 2\delta x^4 - 8x\delta x^3}{(x+\delta x)^4\, x^4}$$

$$\delta y = \frac{\delta x\,(-12x^2\delta x - 8x^3 - 2\delta x^3 - 8x\delta x^2)}{(x+\delta x)^4 x^4}$$

$$\frac{\delta y}{\delta x} = \frac{-12x^2\delta x - 8x^3 - 2\delta x^3 - 8x\delta x^2}{(x+\delta x)^4 x^4}$$

Taking $\lim\limits_{\delta x \to 0}$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{-12x^2\delta x - 8x^3 - 2\delta x^3 - 8x\delta x^2}{(x+\delta x)^4 x^4}$$

$$= \frac{-12x^2(0) - 8x^3 - 2(0)^3 - 8x(0)^2}{(x+0)^4 x^4}$$

$$= \frac{-8x^3}{x^4 x^4}$$

$$= \frac{-8x^3}{x^8}$$

$$= \frac{-8}{x^5}$$

viii) $(x+4)^{1/3}$

Let

$$y = (x+4)^{\frac{1}{3}} \quad \text{---} \quad (1)$$

$$y + \delta y = (x+\delta x+4)^{\frac{1}{3}} \quad \text{---} \quad (2)$$

Subtracting (1) & (2)

$$\cancel{y} + \delta y - \cancel{y} = (x+\delta x+4)^{\frac{1}{3}} - (x+4)^{\frac{1}{3}}$$

$$\delta y = \frac{\left[(x+\delta x+4)^{\frac{1}{3}} - (x+4)^{\frac{1}{3}}\right]\left[(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}\right]}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$\delta y = \frac{(x+\delta x+4)^{\frac{1}{3}\times 3} - (x+4)^{\frac{1}{3}\times 3}}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$\delta y = \frac{\cancel{x} + \delta x + \cancel{4} - \cancel{x} - \cancel{4}}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$\delta y = \frac{\delta x}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$\frac{\delta y}{\delta x} = \frac{1}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

Taking $\lim_{\delta x \to 0} \delta y$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{1}{(x+\delta x+4)^{\frac{2}{3}} + (x+\delta x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$= \frac{1}{(x+0+4)^{\frac{2}{3}} + (x+0+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{\frac{2}{3}}}$$

$$= \frac{1}{(x+4)^{\frac{2}{3}} + (x+4)^{\frac{1}{3}}(x+4)^{\frac{1}{3}} + (x+4)^{2/3}}$$

$$= \frac{1}{(x+4)^{\frac{2}{3}} + (x+4)^{\frac{2}{3}} + (x+4)^{\frac{2}{3}}}$$

$$= \frac{1}{3(x+4)^{\frac{2}{3}}}$$

vi) $x^{3/2}$

Let

$$y = x^{3/2} \quad \text{——} \quad (1)$$

$$y + \delta y = (x+\delta x)^{3/2} \quad \text{——} \quad (2)$$

Subtracting (1) & (2)

$$\cancel{y} + \delta y - \cancel{y} = (x+\delta x)^{3/2} - x^{3/2}$$

$$\delta y = x\left(x + \frac{x\delta x}{x}\right)^{3/2} - x^{3/2}$$

$$\delta y = x^{3/2}\left(1+\frac{\delta x}{x}\right)^{3/2} - x^{3/2}$$

$$\delta y = x^{3/2}\left[(1)^{3/2} + \frac{\frac{3}{2}\left(\frac{\delta x}{x}\right)^1}{1!} + \frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta x}{x}\right)^{3/2}\right] - x^{3/2}$$

$$\delta y = x^{3/2}\left[1 + \frac{3}{2}\left(\frac{\delta x}{x}\right) + \frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta x}{x}\right)^{3/2}\right] - x^{3/2}$$

$$\delta y = \cancel{x^{3/2}} + x^{3/2}\left[\frac{3}{2}\left(\frac{\delta x}{x}\right) + \frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^2 + \dots + \left(\frac{\delta x}{x}\right)^{3/2}\right] \cancel{- x^{3/2}}$$

$$\delta y = x^{3/2}\frac{\delta x}{x'}\left[\frac{3}{2}+\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{\frac{1}{2}}\right]$$

$$\frac{\delta y}{\delta x} = x^{\frac{3}{2}-1}\left[\frac{3}{2}+\frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{\frac{1}{2}}\right]$$

$$\frac{\delta y}{\delta x} = x^{\frac{1}{2}}\left[\frac{3}{2}+\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{\frac{1}{2}}\right]$$

Taking $\lim_{\delta x \to 0}$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0} x^{\frac{1}{2}}\left[\frac{3}{2}+\frac{3}{2}\left(\frac{3}{2}-1\right)\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{\frac{1}{2}}\right]$$

$$= x^{\frac{1}{2}}\left[\frac{3}{2}+\frac{\frac{3}{2}\left(\frac{3}{2}-1\right)}{2!}\left(\frac{0}{x}\right)+\cdots+\left(\frac{0}{x}\right)^{\frac{1}{2}}\right]$$

$$= \frac{3}{2}x^{\frac{1}{2}}$$

x) $x^{5/2}$

Let

$$y = x^{5/2} \quad \text{——— (1)}$$

$$y+\delta y = (x+\delta x)^{5/2} \quad \text{——— (2)}$$

Subtracting ① & ②

$$\cancel{y}+\delta y-\cancel{y}=(x+\delta x)^{5/2}-x^{5/2}$$

$$\delta y = x^{5/2}\left(1+\frac{\delta x}{x}\right)^{5/2}-x^{5/2}$$

$$\delta y = x^{5/2}\left[1+\frac{5}{2}\,\frac{\delta x}{x}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^{2}+\dots+\left(\frac{\delta x}{x}\right)^{5/2}\right]-x^{5/2}$$

$$\delta y = \cancel{x^{5/2}}+x^{5/2}\cdot\left[\frac{5}{2}\,\frac{\delta x}{x}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\left(\frac{\delta x}{x}\right)^{2}+\dots+\left(\frac{\delta x}{x}\right)^{5/2}\right]-\cancel{x^{5/2}}$$

$$\delta y = x^{5/2}\,\frac{\delta x}{x}\left[\frac{5}{2}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\,\frac{\delta x}{x}+\dots+\left(\frac{\delta x}{x}\right)^{3/2}\right]$$

$$\frac{\delta y}{\delta x} = x^{3/2}\left[\frac{5}{2}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\,\frac{\delta x}{x}+\dots+\left(\frac{\delta x}{x}\right)^{3/2}\right]$$

Taking $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0} x^{3/2}\left[\frac{5}{2}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\,\frac{\delta x}{x}+\dots+\left(\frac{\delta x}{x}\right)^{3/2}\right]$$

$$= x^{3/2}\left[\frac{5}{2}+\frac{\frac{5}{2}\left(\frac{5}{2}-1\right)}{2!}\left(\frac{0}{x}\right)+\dots+\left(\frac{0}{x}\right)^{3/2}\right]$$

$$=\frac{5}{2}x^{3/2}$$

$\frac{5}{2}-1$

xii) x^m

let

$$y = x^m \quad \text{--------} \quad (1)$$

$$y + \delta y = (x + \delta x)^m \quad \text{--------} \quad (2)$$

Subtracting (1) & (2)

$$y + \delta y - y = (x + \delta x)^m - x^m$$

$$\delta y = x^m\left(1 + \frac{\delta x}{x}\right)^m - x^m$$

$$\delta y = x^m\left[1 + m\frac{\delta x}{x} + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^m\right] - x^m$$

$$\delta y = \cancel{x^m} + x^m\left[m\frac{\delta x}{x} + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^m\right] \cancel{- x^m}$$

$$\delta y = x^m \frac{\delta x}{x}\left[m + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]$$

$$\frac{\delta y}{\delta x} = x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]$$

Taking $\lim\limits_{\delta x \to 0}$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0} x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{0}{x}\right) + \cdots + \left(\frac{0}{x}\right)^{m-1}\right]$$

$$= x^{m-1}(m)$$

$$= m x^{m-1}$$

xii) $\frac{1}{x^m}$

Let

$$y = \frac{1}{x^m} \text{ ——— } ①$$

$$y + \delta y = \frac{1}{(x+\delta x)^m} \text{ ——— } ②$$

Subtracting ① & ②

$$\cancel{y} + \delta y - \cancel{y} = \frac{1}{(x+\delta x)^m} - \frac{1}{x^m}$$

$$\delta y = \frac{x^m - (x+\delta x)^m}{(x+\delta x)^m x^m}$$

$$\delta y = \frac{x^m - x^m\left(1 + \frac{\delta x}{x}\right)^m}{(x+\delta x)^m x^m}$$

$$\delta y = \frac{x^m - x^m\left[1 + m\frac{\delta x}{x} + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^m\right]}{(x+\delta x)^m x^m}$$

$$\delta y = \frac{\cancel{x^m} - \cancel{x^m} - x^m\left[m\frac{\delta x}{x} + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots \left(\frac{\delta x}{x}\right)^m\right]}{(x+\delta x)^m x^m}$$

$$\delta y = \frac{-x^m\left[m\frac{\delta x}{x} + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^m\right]}{(x+\delta x)^m x^m}$$

$$\delta y = \frac{-x^m \frac{\delta x}{x}\left[m + \frac{m(m-1)}{2!}\frac{\delta x}{x} + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]}{(x+\delta x)^m x^m}$$

$$\frac{\delta y}{\delta x} = \frac{-x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]}{(x+\delta x)^m x^m}$$

Taking $\lim_{\delta x \to 0}$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0}\frac{-x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{m-1}\right]}{(x+\delta x)^m x^m}$$

$$= \frac{-x^{m-1}\left[m + \frac{m(m-1)}{2!}\left(\frac{0}{x}\right) + \cdots + \left(\frac{0}{x}\right)^{m-1}\right]}{(x+0)^m x^m}$$

$$= \frac{-x^{m-1}[m]}{x^m . x^m}$$

$$= \frac{-mx^{m-1}}{x^{2m}} = -mx^{m-1-2m}$$

$$= -mx^{-m-1}$$

xiii) x^{40}

let

$$y = x^{40} \quad \text{---} \quad (1)$$

$$y + \delta y = (x + \delta x)^{40} \quad \text{---} \quad (2)$$

Subtracting (1) & (2)

$$\cancel{y} + \delta y - \cancel{y} = (x + \delta x)^{40} - x^{40}$$

$$\delta y = x^{40}\left(1 + \frac{\delta x}{x}\right)^{40} - x^{40}$$

$$\delta y = x^{40}\left[1 + 40\left(\frac{\delta x}{x}\right) + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^{40}\right] - x^{40}$$

$$\delta y = \cancel{x^{40}} + x^{40}\left[40\left(\frac{\delta x}{x}\right) + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^{40}\right] - \cancel{x^{40}}$$

$$\delta y = x^{40}\left[40\left(\frac{\delta x}{x}\right) + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta x}{x}\right)^{40}\right]$$

$$\boxed{\text{Taking } \lim_{\delta x \to 0}}$$

$$\delta y = x^{40}\frac{\delta x}{x}\left[40 + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{39}\right]$$

$$\frac{\delta y}{\delta x} = x^{40-1}\left[40 + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right) + \cdots + \left(\frac{\delta x}{x}\right)^{39}\right]$$

$$\frac{\delta y}{\delta x} = x^{39}\left[40 + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right) + \dots + \left(\frac{\delta x}{x}\right)^{39}\right]$$

Taking $\lim_{\delta x \to 0}$.

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0} x^{39}\left[40 + \frac{40(40-1)}{2!}\left(\frac{\delta x}{x}\right) + \dots + \left(\frac{\delta x}{x}\right)^{39}\right]$$

$$= x^{39}\left[40 + \frac{40(40-1)}{2!}\left(\frac{0}{x}\right) + \dots + \left(\frac{0}{x}\right)^{39}\right]$$

$$= x^{39}(40)$$

$$= 40x^{39}$$

xiv) x^{-100}

Let

$$y = x^{-100} \quad \text{——— ①}$$

$$y + \delta y = (x+\delta x)^{-100} \quad \text{——— ②}$$

Subtracting ① & ②

$$y + \delta y - y = (x+\delta x)^{-100} - x^{-100}$$

$$\delta y = x^{-100}\left(1+\frac{\delta x}{x}\right)^{-100} - x^{-100}$$

$$\delta y = x^{-100}\left[1+(-100)\frac{\delta x}{x}+\frac{-100(-100-1)}{2!}\left(\frac{\delta x}{x}\right)^2+\cdots+\left(\frac{\delta x}{x}\right)^{-100}\right] - x^{-100}$$

$$\cancel{\delta y = x^{-100}\Big[100}$$

$$\delta y = \cancel{x^{-100}} + x^{-100}\left[-100\frac{\delta x}{x}+\frac{-100(-100-1)}{2!}\left(\frac{\delta x}{x}\right)^2+\cdots+\left(\frac{\delta x}{x}\right)^{-100}\right] \cancel{-x^{-100}}$$

$$\delta y = x^{-100}\frac{\delta x}{x}\left[-100+\frac{-100(-100-1)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{-101}\right]$$

$$\frac{\delta y}{\delta x} = x^{-100-1}\left[-100+\frac{-100(-100-1)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{-101}\right]$$

$$\frac{\delta y}{\delta x} = x^{-101}\left[-100+\frac{-100(-100-1)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{-101}\right]$$

Taking $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0} x^{-101}\left[-100+\frac{-100(-100-1)}{2!}\frac{\delta x}{x}+\cdots+\left(\frac{\delta x}{x}\right)^{-101}\right]$$

$$= x^{-101}[-100]$$

$$= -100x^{-101}$$

FIRST PRINCIPLE

(1) $f(x)$

(2) $f(x+\delta x)$ (3) $f(x+\delta x)-f(x)$

(4) $\dfrac{f(x+\delta x)-f(x)}{\delta x}$

(5) $\lim\limits_{\delta x\to 0}\dfrac{f(x+\delta x)-f(x)}{\delta x}$

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2 i) $\sqrt{x+2}$

Let

$f(x)=\sqrt{x+2}$ —— (1)

$f(x+\delta x)=\sqrt{x+\delta x+2}$ —— (2)

Subtracting (1) & (2)

$f(x+\delta x)-f(x)=\sqrt{x+\delta x+2}-\sqrt{x+2}$

$=\dfrac{\sqrt{x+\delta x+2}-\sqrt{x+2}}{1}\times\dfrac{\sqrt{x+\delta x+2}+\sqrt{x+2}}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$

$=\dfrac{(\sqrt{x+\delta x+2})^2-(\sqrt{x+2})^2}{\sqrt{x+\delta x+2}\sqrt{x+2}}$

$$f(x+\delta x)-f(x)=\frac{\cancel{x}+\delta x+\cancel{2}-\cancel{x}-\cancel{2}}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$f(x+\delta x)-f(x)=\frac{\delta x}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$\frac{f(x+\delta x)-f(x)}{\delta x}=\frac{1}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

Taking $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=\lim_{\delta x\to 0}\frac{1}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$=\frac{1}{\sqrt{x+0+2}+\sqrt{x+2}}$$

$$=\frac{1}{\sqrt{x+2}+\sqrt{x+2}}$$

$$=\frac{1}{2\sqrt{x+2}}$$

$$f(x+\delta x)-f(x)=\frac{\cancel{x}+\delta x+\cancel{2}-\cancel{x}-\cancel{2}}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$f(x+\delta x)-f(x)=\frac{\delta x}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$\frac{f(x+\delta x)-f(x)}{\delta x}=\frac{1}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

Taking $\lim\limits_{\delta x\to 0}$

$$\lim_{\delta x\to 0}\frac{f(x+\delta x)-f(x)}{\delta x}=\lim_{\delta x\to 0}\frac{1}{\sqrt{x+\delta x+2}+\sqrt{x+2}}$$

$$=\frac{1}{\sqrt{x+0+2}+\sqrt{x+2}}$$

$$=\frac{1}{\sqrt{x+2}+\sqrt{x+2}}$$

$$=\frac{1}{2\sqrt{x+2}}$$