Ex # 2.3

Power Rule

$$\frac{d}{dx} x^n = n x^{n-1} \frac{d}{dx}(x)$$

$$\frac{d}{dx}(x) = 1$$

Constant Rule

$$\frac{d}{dx}(c) = 0$$

Product Rule

$$\frac{d}{dx}(u.v) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - (u)\frac{d}{dx}(v)}{v^2}$$

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(1) $x^4 + 2x^3 + x^2$

Differentiate w.r.t 'x'

$$= \frac{d}{dx}(x^4) + 2\frac{d}{dx}(x^3) + \frac{d}{dx}(x^2)$$

$$= 4x^{4-1}\frac{d}{dx}(x) + 2 \times 3x^{3-1}\frac{d}{dx}(x) + 2x^{2-1}\frac{d}{dx}(x)$$

$$= 4x^3(1) + 6x^2(1) + 2x(1)$$

$$= 2x(2x^2 + 3x + 1)$$

(2) $x^{-3} + 2x^{\frac{-3}{2}} + 3$

Diff — w.r.t. x

$$= \frac{d}{dx}(x^{-3}) + 2\frac{d}{dx}(x^{\frac{-3}{2}}) + \frac{d}{dx}(3)$$

$$= -3x^{-3-1}\frac{d}{dx}(x) + 2x\frac{-3}{2}x^{\frac{-3}{2}-1}\frac{d}{dx}(x) + 0$$

$$= -3x^{-4}(1) - 3x^{-\frac{5}{2}}(1)$$

$$= -3x^{-4} - 3x^{-\frac{5}{2}}$$

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(3) $\frac{a+x}{a-x}$

Diff — w.r.t. 'x'

$$= \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$= \frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2}$$

$$= \frac{(a-x)\left[\frac{d}{dx}(a) + \frac{d}{dx}(x)\right] - (a+x)\left[\frac{d}{dx}(a) - \frac{d}{dx}(x)\right]}{(a-x)^2}$$

$$= \frac{(a-x)(0+1) - (a+x)(0-1)}{(a-x)^2}$$

$$= \frac{a-x + a+x}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2}$$

(4) $\dfrac{2x-3}{2x+1}$

Diff w.r.t 'x'

$$= \frac{d}{dx}\left(\frac{2x-3}{2x+1}\right)$$

$$= \frac{(2x+1)\frac{d}{dx}(2x-3) - (2x-3)\frac{d}{dx}(2x+1)}{(2x+1)^2}$$

$$= \frac{(2x+1)\left[\frac{2d}{dx}(x) - \frac{d}{dx}(3)\right] - (2x-3)\left[\frac{2d}{dx}(x) + \frac{d}{dx}(1)\right]}{(2x+1)^2}$$

$$= \frac{(2x+1)(2(1)-0) - (2x-3)(2(1)+0)}{(2x+1)^2}$$

$$= \frac{(2x+1)(2) - (2x-3)(2)}{(2x+1)^2}$$

$$= \frac{4x+2-4x+6}{(2x+1)^2} = \frac{8}{(2x+1)^2}$$

(5) $(x-5)(3-x)$

$= 3x - x^2 - 15 + 5x$

$= 8x - x^2 - 15$

Diff - w.r.t 'x'

$= 8\frac{d}{dx}(x) - \frac{d}{dx}(x^2) - \frac{d}{dx}(15)$

$= 8(1) - 2x^{2-1}\frac{d}{dx}(x) - 0$

$= 8 - 2x(1)$

$= 8 - 2x$

(6) $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$= (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x} \times \frac{1}{\sqrt{x}}$

$= x + \frac{1}{x} - 2$

Diff - w.r.t 'x'

$= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-1}) - \frac{d}{dx}(2)$

$$= (1) + (-1)x^{-1-1}\frac{d}{dx}(x) - 0$$

$$= 1 - x^{-2}(1)$$

$$= 1 - \frac{1}{x^2}$$

$$= \frac{x^2 - 1}{x^2}$$

(7) $$\frac{(1+\sqrt{x})(x - x^{3/2})}{\sqrt{x}}$$

$$= \frac{(1+\sqrt{x})x(1-\sqrt{x})}{\sqrt{x}}$$

$$= \frac{x((1)^2 - (\sqrt{x})^2)}{\sqrt{x}}$$

$$= \sqrt{x}(1-x)$$

$$= \sqrt{x} - x\sqrt{x}$$

$$= x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

Diff - w.r.t 'x'

$$= \frac{d}{dx}x^{\frac{1}{2}} - \frac{d}{dx}x^{\frac{3}{2}}$$

$$= \frac{1}{2}x^{\frac{1}{2}-1}\frac{d}{dx}(x) - \frac{3}{2}x^{\frac{3}{2}-1}\frac{d}{dx}(x)$$

$x^{\frac{3}{2}} = x^{1+\frac{1}{2}}$

$= x^1 \cdot x^{\frac{1}{2}}$

$= x\sqrt{x}$

$\frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{1}$

$$= \frac{1}{2} x^{-\frac{1}{2}}(1) - \frac{3}{2} x^{\frac{1}{2}}(1)$$

$$= \frac{1}{2x^{\frac{1}{2}}} - \frac{3}{2} x^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1}{x^{\frac{1}{2}}} - 3x^{\frac{1}{2}}\right)$$

$$= \frac{1}{2} \cdot \frac{1 - 3x^{\frac{1}{2}} \times x}{x^{\frac{1}{2}}}$$

$$= \frac{1 - 3x}{2x^{\frac{1}{2}}}$$

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(8) $\dfrac{(x^2+1)^2}{x^2-1}$

Diff — w.r.t 'x'

$$= \frac{d}{dx} \frac{(x^2+1)^2}{x^2-1}$$

$$= \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2 \frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1).2\frac{d}{dx}(x^2+1)^{2-1}\frac{d}{dx}(x^2+1) - (x^2+1)^2\left[\frac{d}{dx}x^2 - \frac{d}{dx}(1)\right]}{(x^2-1)^2}$$

$$= \frac{2(x^2-1)(x^2+1)\left[\frac{d}{dx}(x^2)+\frac{d}{dx}(1)\right]-(x^2+1)^2(2x-0)}{(x^2-1)^2}$$

$$= \frac{2(x^2-1)(x^2+1)(2x+0)-2x(x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)\left[2(x^2-1)-(x^2+1)\right]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)\left[2x^2-2-x^2-1\right]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

(9) $\frac{x^2+1}{x^2-3}$

Diff w.r.t. 'x'

$$= \frac{d}{dx}\left(\frac{x^2+1}{x^2-3}\right)$$

$$= \frac{(x^2-3)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-3)}{(x^2-3)^2}$$

$$= \frac{(x^2-3)(2x+0) - (x^2+1)(2x-0)}{(x^2-3)^2}$$

$$= \frac{2x(x^2-3-x^2-1)}{(x^2-3)^2}$$

$$= \frac{2x(-4)}{(x^2-3)^2}$$

$$= \frac{-8x}{(x^2-3)^2}$$

(10) $\dfrac{\sqrt{1+x}}{\sqrt{1-x}}$

$$= \sqrt{\frac{1+x}{1-x}}$$

Diff - w.r.t 'x'

$$= \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \frac{(1-x)\frac{d}{dx}(1+x) - (1+x)\frac{d}{dx}(1-x)}{(1-x)^2}$$

$$= \frac{1}{2}\frac{(1+x)^{-\frac{1}{2}}}{(1-x)^{-\frac{1}{2}}} \frac{(1-x)(0+1) - (1+x)(0-1)}{(1-x)^2}$$

$$= \frac{1}{2}\frac{(1+x)^{-\frac{1}{2}}}{(1-x)^{-\frac{1}{2}+2}}(1-x+1+x)$$

$$= \frac{1}{2}\frac{2}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

$$= \frac{1}{(1-x)^{\frac{3}{2}}(1+x)^{\frac{1}{2}}}$$

$$= \frac{1}{(1-x)(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}$$

$$= \frac{1}{(1-x)(1-x^2)^{\frac{1}{2}}}$$

$$= \frac{1}{(1-x)\sqrt{1-x^2}}$$

(11) $\dfrac{2x-1}{\sqrt{x^2+1}}$

Differentiate w.r.t. 'x'

$$= \frac{d}{dx}\left(\frac{2x-1}{\sqrt{x^2+1}}\right)$$

$$= \frac{\sqrt{x^2+1}\,\frac{d}{dx}(2x-1) - (2x-1)\,\frac{d}{dx}(x^2+1)^{\frac{1}{2}}}{\left(\sqrt{x^2+1}\right)^2}$$

$$= \frac{\sqrt{x^2+1}\left[\frac{d}{dx}(x) - \frac{d}{dx}(1)\right] - (2x-1)\frac{1}{2}(x^2+1)^{\frac{1}{2}-1}\frac{d}{dx}(x^2+1)}{x^2+1}$$

$$= \frac{\sqrt{x^2+1}\left[2(1)-0\right] - (2x-1)\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x+0)}{(x^2+1)}$$

$$= \frac{2\sqrt{x^2+1} - \dfrac{x(2x-1)}{\sqrt{x^2+1}}}{(x^2+1)}$$

$$= \frac{\dfrac{2\left(\sqrt{x^2+1}\right)^2 - 2x^2 + x}{\sqrt{x^2+1}}}{(x^2+1)}$$

$$= \frac{2x^2+2-2x^2+x}{(x^2+1)\sqrt{x^2+1}}$$

$$= \frac{x+2}{(x^2+1)(x^2+1)^{\frac{1}{2}}}$$

$$= \frac{x+2}{(x^2+1)^{\frac{3}{2}}}$$

(12) $\sqrt{\frac{a-x}{a+x}}$

Differentiate w.r.t. 'x'

$$= \frac{d}{dx}\left(\frac{a-x}{a+x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{a-x}{a+x}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{a-x}{a+x}\right)$$

$$= \frac{1}{2}\left(\frac{a-x}{a+x}\right)^{-\frac{1}{2}} \cdot \frac{(a+x)\frac{d}{dx}(a-x) - (a-x)\frac{d}{dx}(a+x)}{(a+x)^2}$$

$$= \frac{1}{2}\frac{(a-x)^{-\frac{1}{2}}}{(a+x)^{-\frac{1}{2}}} \cdot \frac{(a+x)(0-1)-(a-x)(0+1)}{(a+x)^2}$$

$$= \frac{1}{2}\frac{(a+x)(-1)-(a-x)(1)}{(a-x)^{\frac{1}{2}}(a+x)^{2-\frac{1}{2}}}$$

$$= \frac{1}{2} \frac{-a-x-a+x}{(a-x)^{\frac{1}{2}}(a+x)^{3/2}}$$

$$= \frac{1}{2} \frac{-2a}{(a-x)^{\frac{1}{2}}(a+x)(a+x)^{\frac{1}{2}}}$$

$$= \frac{-a}{(a+x)[(a-x)(a+x)]^{\frac{1}{2}}}$$

$$= \frac{-a}{(a+x)\sqrt{a^2-x^2}}$$

(13) $\dfrac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

$= \sqrt{\dfrac{x^2+1}{x^2-1}}$

Differentiate w.r.t. 'x'

$= \dfrac{d}{dx}\left(\dfrac{x^2+1}{x^2-1}\right)^{\frac{1}{2}}$

$= \dfrac{1}{2}\left(\dfrac{x^2+1}{x^2-1}\right)^{\frac{1}{2}-1} \cdot \dfrac{d}{dx}\left(\dfrac{x^2+1}{x^2-1}\right)$

$= \dfrac{1}{2}\left(\dfrac{x^2+1}{x^2-1}\right)^{-\frac{1}{2}} \dfrac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \dfrac{1}{2}\left(\dfrac{x^2+1}{x^2-1}\right)^{-\frac{1}{2}} \dfrac{(x^2-1)(2x+0) - (x^2+1)(2x-0)}{(x^2-1)^2}$

$= \dfrac{1}{2} \dfrac{(x^2+1)^{-\frac{1}{2}}}{(x^2-1)^{-\frac{1}{2}}} \dfrac{2x(x^2-1-x^2-1)}{(x^2-1)^2}$

$= \dfrac{1}{2} \dfrac{2x(-2)}{(x^2+1)^{\frac{1}{2}}(x^2-1)^{2-\frac{1}{2}}}$

$= \dfrac{-2x}{(x^2+1)^{\frac{1}{2}}(x^2-1)^{\frac{3}{2}}}$

$$= \frac{-2x}{(x^2+1)^{\frac{1}{2}}(x^2-1)(x^2-1)^{\frac{1}{2}}}$$

$$= \frac{-2x}{(x^2-1)(x^2+1)^{\frac{1}{2}}(x^2-1)^{\frac{1}{2}}}$$

$$= \frac{-2x}{(x^2-1)\sqrt{(x^2)^2-(1)^2}}$$

$$= \frac{-2x}{(x^2-1)\sqrt{x^4-1}}$$

(14) $\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$= \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \times \dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

$= \dfrac{(\sqrt{1+x}-\sqrt{1-x})^2}{(\sqrt{1+x})^2-(\sqrt{1-x})^2}$

$= \dfrac{(\sqrt{1+x})^2+(\sqrt{1-x})^2-2\sqrt{1+x}\sqrt{1-x}}{1+x-1+x}$

$= \dfrac{1+x+1-x-2\sqrt{1-x^2}}{2x}$

$= \dfrac{2-2\sqrt{1-x^2}}{2x}$

$= \dfrac{2(1-\sqrt{1-x^2})}{2x}$

$= \dfrac{1-\sqrt{1-x^2}}{x}$

Differentiate w.r.t. 'x'

$$= \frac{d}{dx}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$$

$$= \frac{x\frac{d}{dx}(1-(1-x^2)^{\frac{1}{2}}) - (1-\sqrt{1-x^2})\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\left[0-\frac{1}{2}(1-x^2)^{\frac{1}{2}-1}\frac{d}{dx}(1-x^2)\right] - (1-\sqrt{1-x^2})(1)}{x^2}$$

$$= \frac{x\left[-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(0-2x)\right] - (1-\sqrt{1-x^2})}{x^2}$$

$$= \frac{\frac{x^2}{\sqrt{1-x^2}} - 1+\sqrt{1-x^2}}{x^2}$$

$$= \frac{x^2 - \sqrt{1-x^2} + (\sqrt{1-x^2})^2}{\sqrt{1-x^2}\,x^2}$$

$$= \frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{x^2\sqrt{1-x^2}}$$

$$= \frac{1 - \sqrt{1-x^2}}{x^2\sqrt{1-x^2}}$$

(15) $$\frac{x\sqrt{a+x}}{\sqrt{a-x}} = x\sqrt{\frac{a+x}{a-x}}$$

Differentiate w.r.t. 'x'

$$= \frac{d}{dx}\, x\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}$$

$$= x\frac{d}{dx}\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}} + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}\frac{d}{dx}(x)$$

$$= x\,\frac{1}{2}\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{a+x}{a-x}\right) + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}(1)$$

$$= \frac{x}{2}\left(\frac{a+x}{a-x}\right)^{-\frac{1}{2}}\frac{(a-x)\frac{d}{dx}(a+x) - (a+x)\frac{d}{dx}(a-x)}{(a-x)^2} + \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}$$

$$= \frac{x}{2}\frac{(a+x)^{-\frac{1}{2}}}{(a-x)^{-\frac{1}{2}}}\frac{(a-x)(0+1) + (a+x)(0+1)}{(a-x)^2} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{x}{2}\frac{(a+x)^{-\frac{1}{2}}[a-x+a+x]}{(a-x)^{-\frac{1}{2}+2}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{x}{x} \cdot \frac{2a}{(a-x)^{\frac{3}{2}}(a+x)^{\frac{1}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{xa}{(a-x)^{1}(a-x)^{\frac{1}{2}}(a+x)^{\frac{1}{2}}} + \frac{(a+x)^{\frac{1}{2}}}{(a-x)^{\frac{1}{2}}}$$

$$= \frac{xa + (a-x)[(a+x)^{\frac{1}{2}}]^{x}}{(a-x)(a-x)^{\frac{1}{2}}(a+x)^{\frac{1}{2}}}$$

$$= \frac{xa + (a-x)(a+x)}{(a-x)^{\frac{3}{2}}(a+x)^{\frac{1}{2}}}$$

$$= \frac{xa + a^2 - x^2}{(a-x)^{3/2}(a+x)^{3/2}}$$

(16) $y = \sqrt{x} - \frac{1}{\sqrt{x}}$

Diff — w.r.t 'x'

$$\frac{d}{dx}y = \frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(x^{\frac{-1}{2}})$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}\frac{d}{dx}(x) - \left(\frac{-1}{2}\right)x^{\frac{-1}{2}-1}\frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}(1) + \frac{1}{2}x^{-\frac{3}{2}}(1)$$

$$\frac{dy}{dx} = \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x^{3/2}}$$

$$\frac{dy}{dx} = \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{2x\,x^{\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{x+1}{2x\,x^{\frac{1}{2}}} \Rightarrow$$

$$2x\frac{dy}{dx} = \frac{x+1}{x^{\frac{1}{2}}}$$

$$2x\frac{dy}{dx} = \frac{x}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}$$

$$2x\frac{dy}{dx} = x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}$$

$$2x\frac{dy}{dx} + y = \sqrt{x} + \frac{1}{\sqrt{x}} + y \quad \text{Adding } y,$$

$$2x\frac{dy}{dx} + y = \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$2x\frac{dy}{dx} + y = 2\sqrt{x}$$

Hence Proved

Exp

(17) $y = x^4 + 2x^2 + 2$

Diff — w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + 2\frac{d}{dx}(x^2) + \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = 4x^{4-1}\frac{d}{dx}(x) + 2 \times 2x^{2-1}\frac{d}{dx}(x) + 0$$

$$\frac{dy}{dx} = 4x^3(1) + 4x(1)$$

$$\frac{dy}{dx} = 4x(x^2+1)$$

$$\frac{dy}{dx} = 4x\sqrt{(x^2+1)^2}$$

$$\frac{dy}{dx} = 4x\sqrt{x^4+2x^2+1}$$

$$\frac{dy}{dx} = 4x\sqrt{x^4+2x^2+2-1}$$

$$\frac{dy}{dx} = 4x\sqrt{y-1}$$

Hence Proved