Unit 1 Real Numbers

EXERCISE 1.1

1. Identify each of the following as a rational or irrational number:

(i) 2.353535 (ii) $0.\overline{6}$ (iii) 2.236067... (iv) $\sqrt{7}$

(v) e (vi) π (vii) $5+\sqrt{11}$ (viii) $\sqrt{3}+\sqrt{13}$

(ix) $\frac{15}{4}$ (x) $(2-\sqrt{2})(2+\sqrt{2})$

Solution

(i) Rational (ii) Rational (iii) Irrational (iv) Irrational (v) Irrational

(vi) Irrational (vii) Irrational (viii) Irrational (ix) Rational (x) rational

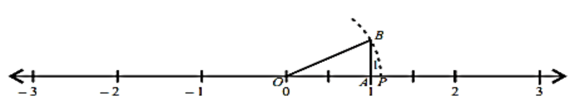
2. Represent the following numbers on number line:

(i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $4\frac{1}{3}$

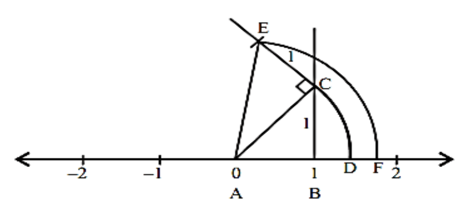
(iv) $-2\frac{1}{7}$ (v) $\frac{5}{8}$ (vi) $2\frac{3}{4}$

Solution

(i)



(ii)



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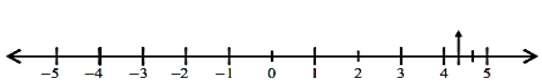

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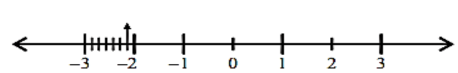

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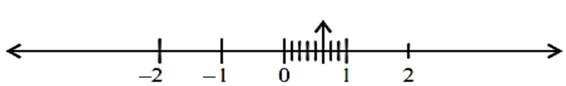

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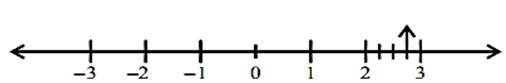


3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i) $0.\overline{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

$\boldsymbol{x = 0.\overline{4}}$ $x = 0.4444 \ldots$ $10x = 10(0.4444 \ldots)$ $10x = 4.4444 \ldots$ $10x - x = (4.4444 \ldots) - (0.4444 \ldots)$ $9x = 4 \Rightarrow \boldsymbol{x = \frac{4}{9}}$	$\boldsymbol{x = 0.\overline{37}}$ $x = 0.3737 \ldots$ $100x = 100(0.3737 \ldots)$ $100x = 37.3737 \ldots$ $100x - x = (37.3737 \ldots) - (0.3737 \ldots)$ $99x = 37 \Rightarrow \boldsymbol{x = \frac{37}{99}}$

$$\boldsymbol{x = 0.\overline{21}}$$

$$x = 0.2121 \ldots$$

$$100x = 100(0.2121 \ldots)$$

$$100x = 21.2121 \ldots$$

$$100x - x = (21.2121 \ldots) - (0.2121 \ldots)$$

$$99x = 21 \Rightarrow \boldsymbol{x = \frac{21}{99}}$$

4. Name the property used in the following:

(i) $(a+4)+b = a+(4+b)$ (ii) $\sqrt{2}+\sqrt{3} = \sqrt{3}+\sqrt{2}$

(iii) $x - x = 0$ (iv) $a(b+c) = ab+ac$

(v) $16+0 = 16$ (vi) $100 \times 1 = 100$

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$ (viii) $ab = ba$

Solution

(i) Associative property over addition
(ii) Commutative property over addition
(iii) Additive inverse
(iv) Left distributive property
(v) Additive identity
(vi) Multiplicative identity
(vii) Associative property under multiplication
(viii) Commutative property under multiplication

5. Name the property used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$

(ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

(iii) If $a < b$ then $a + c < b + c$

(iv) If $ac < bc$ and $c > 0$ then $a < b$

(v) If $ac < bc$ and $c < 0$ then $a > b$

(vi) Either $a > b$ or $a = b$ or $a < b$

Solution

(i) Additive property
(ii) Reciprocal property
(iii) Additive property
(iv) Multiplicative property
(v) Multiplicative property
(vi) Trichotomy property

6. Insert two rational numbers between:

(i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4 (iii) $\frac{3}{5}$ and $\frac{4}{5}$

Solution

i. $q_1 = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{7}{12}\right) = \frac{7}{24}$ and $q_2 = \frac{1}{2}\left(\frac{7}{24} + \frac{1}{4}\right) = \frac{1}{2}\left(\frac{13}{24}\right) = \frac{13}{48}$

Hence required rational are $\mathbf{\frac{7}{24}, \frac{13}{48}}$

ii. $q_1 = \frac{1}{2}(3 + 4) = \frac{7}{2}$ and $q_2 = \frac{1}{2}\left(\frac{7}{2} + 4\right) = \frac{1}{2}\left(\frac{15}{2}\right) = \frac{15}{4}$

Hence required rational are $\mathbf{\frac{7}{2}, \frac{15}{4}}$

iii. $q_1 = \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{1}{2}\left(\frac{7}{5}\right) = \frac{7}{10}$ and $q_2 = \frac{1}{2}\left(\frac{7}{10} + \frac{4}{5}\right) = \frac{1}{2}\left(\frac{15}{10}\right) = \frac{3}{4}$

Hence required rational are $\mathbf{\frac{7}{10}, \frac{3}{4}}$

EXERCISE 1.2

1. Rationalize the denominator of following:

(i) $\dfrac{13}{4+\sqrt{3}}$ (ii) $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ (iii) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

(iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (vi) $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solution

i. $\dfrac{\mathbf{13}}{\mathbf{4+\sqrt{3}}} = \dfrac{13}{4+\sqrt{3}} \times \dfrac{4-\sqrt{3}}{4-\sqrt{3}} = \dfrac{13(4-\sqrt{3})}{(4)^2-(\sqrt{3})^2} = \dfrac{13(4-\sqrt{3})}{16-3} = \dfrac{13(4-\sqrt{3})}{13} = \mathbf{4-\sqrt{3}}$

ii. $\dfrac{\mathbf{\sqrt{2}+\sqrt{5}}}{\mathbf{\sqrt{3}}} = \dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{(\sqrt{2}+\sqrt{5})\sqrt{3}}{\sqrt{3}.\sqrt{3}} = \dfrac{\sqrt{2}.\sqrt{3}+\sqrt{5}.\sqrt{3}}{3} = \dfrac{\mathbf{\sqrt{6}+\sqrt{15}}}{\mathbf{3}}$

iii. $\dfrac{\mathbf{\sqrt{2}-1}}{\mathbf{\sqrt{5}}} = \dfrac{\sqrt{2}-1}{\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{(\sqrt{2}-1)\sqrt{5}}{\sqrt{5}.\sqrt{5}} = \dfrac{\sqrt{2}.\sqrt{5}-1.\sqrt{5}}{5} = \dfrac{\mathbf{\sqrt{10}-\sqrt{5}}}{\mathbf{5}}$

iv. $\dfrac{\mathbf{6-4\sqrt{2}}}{\mathbf{6+4\sqrt{2}}} = \dfrac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \dfrac{6-4\sqrt{2}}{6-4\sqrt{2}} = \dfrac{(6-4\sqrt{2})^2}{(6)^2-(4\sqrt{2})^2} = \dfrac{(6)^2+(4\sqrt{2})^2-2(6)(4\sqrt{2})}{36-16(2)}$

$= \dfrac{36+32-48\sqrt{2}}{36-32} = \dfrac{68-48\sqrt{2}}{4} = \mathbf{17-12\sqrt{2}}$

v. $\dfrac{\mathbf{\sqrt{3}-\sqrt{2}}}{\mathbf{\sqrt{3}+\sqrt{2}}} = \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \dfrac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = \dfrac{(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})}{3-2}$

$= \dfrac{3+2-2\sqrt{6}}{1} = \mathbf{5-2\sqrt{6}}$

vi. $\dfrac{\mathbf{4\sqrt{3}}}{\mathbf{\sqrt{7}+\sqrt{5}}} = \dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2} = \dfrac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$

$= \dfrac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2} = \mathbf{2\sqrt{3}}(\sqrt{7}-\sqrt{5})$

2. Simplify the following:

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ (ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$ (iii) $(0.027)^{-\frac{1}{3}}$

(iv) $\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}}$ (v) $\frac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(5)^{2n+3} - (25)^{n+1}}$

(vi) $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$ (vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

(viii) $\frac{3^{n} \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$ (ix) $\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^{n} - 2^{n} \times 5^{n}}$

Solution

i. $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}} = \frac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}} = \frac{2^3}{3^3} = \mathbf{\frac{8}{27}}$

ii. $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27} = \left(\frac{4}{3}\right)^{2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27} = \frac{4^2}{3^2} \times \frac{9^3}{4^3} \times \frac{16}{27} = \frac{16\times729\times16}{9\times64\times27} = \mathbf{12}$

iii. $(0.027)^{-\frac{1}{3}} = \left(\frac{27}{1000}\right)^{-\frac{1}{3}} = \left(\frac{1000}{27}\right)^{\frac{1}{3}} = \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}} = \frac{10^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}} = \mathbf{\frac{10}{3}}$

iv. $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}} = \left(\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}\right)^{\frac{1}{7}} = (x^{14} \times y^{7} \times z^{28})^{\frac{1}{7}}$

$= x^{14\times\frac{1}{7}} \times y^{7\times\frac{1}{7}} \times z^{28\times\frac{1}{7}} = \mathbf{x^2yz^4}$

v. $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}} = \frac{5.(5^2)^{n+1} - 5^2.(5)^{2n}}{5.(5)^{2n+3} - (5^2)^{n+1}} = \frac{5.5^{2n+2} - 5^2.5^{2n}}{5.5^{2n+3} - 5^{2n+2}} = \frac{5^{2n+3} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}}$

$= \frac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)} = \frac{5-1}{25-1} = \frac{4}{24} = \mathbf{\frac{1}{6}}$

vi. $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}} = \frac{(2^4)^{x+1} + 20.(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}} = \frac{2^{4x+4} + 20.2^{4x}}{2^{x-3} \times 2^{3x+6}} = \frac{2^{4x+4} + 20.2^{4x}}{2^{4x+3}}$

$= \frac{2^{4x}(2^4+20)}{2^{4x}.2^3} = \frac{(16+20)}{2^3} = \frac{36}{8} = \mathbf{\frac{9}{2}}$

vii. $\mathbf{(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}} = \frac{(64)^{-\frac{2}{3}}}{(9)^{-\frac{3}{2}}} = \frac{(9)^{\frac{3}{2}}}{(64)^{\frac{2}{3}}} = \frac{(3^2)^{\frac{3}{2}}}{(4^3)^{\frac{2}{3}}} = \frac{3^3}{4^2} = \mathbf{\frac{27}{16}}$

viii. $\mathbf{\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}} = \frac{3^n \times (3^2)^{n+1}}{3^{n-1} \times (3^2)^{n-1}} = \frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}} = \frac{3^{3n+2}}{3^{3n-3}} = 3^{3n+2-3n+3} = 3^5 = \mathbf{243}$

ix. $\mathbf{\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^n \times 5^n}}$ =???

$\mathbf{\frac{5^{n+3}-6.5^{n+1}}{9\times5^n-2^n\times5^n}}$ **wrong statement**	$\mathbf{\frac{5^{n+3}-6.5^{n+1}}{9\times5^n-2^n\times5^n}}$ **according to book**
$\mathbf{\frac{5^{n+3}-6.5^{n+1}}{9\times5^n-2^2\times5^n}}$ **right statement**	$= \frac{5^{n+1}(5^2-6)}{5^n(9-2^n)} = \frac{5(5^2-6)}{(9-2^n)}$
$= \frac{5^n(5^3-6.5^1)}{5^n(9-2^2)}$	$= \frac{5(25-6)}{(9-2^n)} = \frac{5(19)}{(9-2^n)} = \frac{5(19)}{(9-2^2)}$; $n = 2$
$= \frac{125-30}{9-4} = \frac{95}{5} = \mathbf{19}$	$= \frac{5(19)}{9-4} = \frac{5(19)}{5} = \mathbf{19}$

3. If $x = 3 + \sqrt{8}$ then find the value of:

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$ (iii) $x^2 + \frac{1}{x^2}$

(iv) $x^2 - \frac{1}{x^2}$ (v) $x^4 + \frac{1}{x^4}$ (vi) $\left(x - \frac{1}{x}\right)^2$

Solution

$x = 3 + \sqrt{8} \Rightarrow \frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2} = \frac{3-\sqrt{8}}{9-8} = 3 - \sqrt{8}$

Hence $\boldsymbol{x = 3 + \sqrt{8}}$ and $\boldsymbol{\frac{1}{x} = 3 - \sqrt{8}}$

i. $x + \frac{1}{x} = (3 + \sqrt{8}) + (3 - \sqrt{8}) = \mathbf{6}$

ii. $x - \frac{1}{x} = (3 + \sqrt{8}) - (3 - \sqrt{8}) = \mathbf{2\sqrt{8}}$

iii. $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = (6)^2 - 2 = 36 - 2 = \mathbf{34}$

iv. $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = (6)(2\sqrt{8}) = \mathbf{12\sqrt{8}}$

v. $x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = (34)^2 - 2 = 1156 - 2 = \mathbf{1154}$

vi. $\left(x - \frac{1}{x}\right)^2 = (2\sqrt{8})^2 = 4 \times 8 = \mathbf{32}$

4. Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

Solution

$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = p + q\sqrt{2}$

$\frac{32-24\sqrt{2}-12\sqrt{2}+18}{(4)^2-(3\sqrt{2})^2} = p + q\sqrt{2}$

$\frac{50-36\sqrt{2}}{16-18} = p + q\sqrt{2}$

$\frac{50-36\sqrt{2}}{-2} = p + q\sqrt{2}$

$-25 + 18\sqrt{2} = p + q\sqrt{2}$

Hence $\boldsymbol{p = -25}$ and $\boldsymbol{q = 18}$

5. Simplify the following:

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

(ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Solution

i. $\dfrac{(\mathbf{25})^{\frac{3}{2}}\times(\mathbf{243})^{\frac{3}{5}}}{(\mathbf{16})^{\frac{5}{4}}\times(\mathbf{8})^{\frac{4}{3}}}=\dfrac{(5^2)^{\frac{3}{2}}\times(3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}}=\dfrac{5^3\times3^3}{2^5\times2^4}=\dfrac{5^3\times3^3}{2^9}=\dfrac{125\times27}{512}=\dfrac{\mathbf{3375}}{\mathbf{512}}$

ii. $\dfrac{\mathbf{54}\times\sqrt[3]{(\mathbf{27})^{2x}}}{\mathbf{9}^{x+1}+\mathbf{216}(\mathbf{3}^{2x-1})}=\dfrac{54\times(27)^{\frac{2x}{3}}}{9^{x+1}+216(3^{2x-1})}=\dfrac{54\times(3^3)^{\frac{2x}{3}}}{(3^2)^{x+1}+216(3^{2x-1})}=\dfrac{54\times3^{2x}}{3^{2x+2}+216(3^{2x-1})}$

$=\dfrac{54\times3^{2x}}{3^{2x}\left(3^2+216(3^{-1})\right)}=\dfrac{54}{\left(3^2+\frac{216}{3}\right)}=\dfrac{54}{9+72}=\dfrac{54}{81}=\dfrac{\mathbf{2}}{\mathbf{3}}$

iii. $\sqrt{\dfrac{(\mathbf{216})^{\frac{2}{3}}\times(\mathbf{25})^{\frac{1}{2}}}{(\mathbf{0.04})^{-\frac{3}{2}}}}=\left(\dfrac{(6^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}\right)^{\frac{1}{2}}=\left(\dfrac{6^2\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}\right)^{\frac{1}{2}}=\left(\dfrac{6^2\times5}{(25)^{\frac{3}{2}}}\right)^{\frac{1}{2}}=\left(\dfrac{6^2\times5}{(5^2)^{\frac{3}{2}}}\right)^{\frac{1}{2}}$

$=\left(\dfrac{6^2\times5}{5^3}\right)^{\frac{1}{2}}=\left(\dfrac{6^2}{5^2}\right)^{\frac{1}{2}}=\dfrac{\mathbf{6}}{\mathbf{5}}$

iv. $\left(\boldsymbol{a}^{\frac{1}{3}}+\boldsymbol{b}^{\frac{2}{3}}\right)\times\left(\boldsymbol{a}^{\frac{2}{3}}-\boldsymbol{a}^{\frac{1}{3}}\boldsymbol{b}^{\frac{2}{3}}+\boldsymbol{b}^{\frac{4}{3}}\right)$

$=\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

$=\left(a^{\frac{1}{3}}a^{\frac{2}{3}}-a^{\frac{1}{3}}a^{\frac{1}{3}}b^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{4}{3}}+b^{\frac{2}{3}}a^{\frac{2}{3}}-b^{\frac{2}{3}}a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{2}{3}}b^{\frac{4}{3}}\right)$

$=\left(a^{\frac{1}{3}+\frac{2}{3}}-a^{\frac{1}{3}+\frac{1}{3}}b^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{4}{3}}+a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}+\frac{2}{3}}+b^{\frac{2}{3}+\frac{4}{3}}\right)$

$=\left(a^{\frac{3}{3}}-\cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}}+\cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}}+\cancel{a^{\frac{2}{3}}b^{\frac{2}{3}}}-\cancel{a^{\frac{1}{3}}b^{\frac{4}{3}}}+b^{\frac{6}{3}}\right)$

$=\boldsymbol{a}+\boldsymbol{b}^{\mathbf{2}}$

EXERCISE 1.3

1. The sum of three consecutive integers is forty-two, find the three integers.

Solution

Consider three consecutive integers are x , $(x+1)$ and $(x+2)$

$(x)+(x+1)+(x+2)=42$

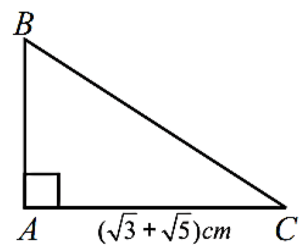
$3x+3=42$

$3x=39$

$x=13$

Hence the three consecutive integers are **13**, **14**, and **15**.

2. The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $(\sqrt{3}+\sqrt{5})$cm. The area of ΔABC is $(1+\sqrt{15})$ cm^2. Find the length $\overline{AB}$ in the form $(a\sqrt{3}+b\sqrt{5})$ cm, where a and b are integers.



Solution

Length of $\overline{AC}=\left(\sqrt{3}+\sqrt{5}\right)$ cm

Area of $\Delta ABC=\left(1+\sqrt{15}\right)$ cm^2

Area of $\Delta ABC=\frac{1}{2}\times$ base $\times$ height

$\left(1+\sqrt{15}\right)=\frac{1}{2}\times\left(\sqrt{3}+\sqrt{5}\right)\times\overline{AB}$

$\left(2+2\sqrt{15}\right)=\left(\sqrt{3}+\sqrt{5}\right)\times\overline{AB}$

$\overline{AB}=\frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}}=\frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}}\times\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{45}-2\sqrt{75}}{\left(\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}$

$\overline{AB}=\frac{2\sqrt{3}-2\sqrt{5}+6\sqrt{5}-10\sqrt{3}}{3-5}=\frac{-8\sqrt{3}+4\sqrt{5}}{-2}=\mathbf{\left(4\sqrt{3}-2\sqrt{5}\right)}$

3. A rectangle has sides of length $2+\sqrt{18}$ m and $\left(5-\frac{4}{\sqrt{2}}\right)$m. Express the area of the rectangle in the form $a+b\sqrt{2}$, where a and b are integers.

Solution

Area $=L\times W=\left(2+\sqrt{18}\right)\times\left(5-\frac{4}{\sqrt{2}}\right)=10-\frac{8}{\sqrt{2}}+5\sqrt{18}-\sqrt{18}\left(\frac{4}{\sqrt{2}}\right)$

Area $=10-\frac{4\times2}{\sqrt{2}}+5\sqrt{9\times2}-4\sqrt{\frac{18}{2}}=10-4\sqrt{2}+5\times3\sqrt{2}-4\sqrt{9}$

Area $=10-4\sqrt{2}+15\sqrt{2}-12=\mathbf{\left(11\sqrt{2}-2\right)}$ m^2

4. Find two numbers whose sum is 68 and difference is 22.

Solution

Let x equal the first number and y equal the second number. Then

According to condition: $x + y = 68$ and $x - y = 22$

$x + y = 68$ $x - y = 22$ adding both $\boldsymbol{x = 45}$	$x + y = 68$ $-x \mp y = 22$ subtracting both $\boldsymbol{y = 23}$

5. The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperature as high as $48℃$. By using the formula, $(°F = \frac{9}{5}°C + 32)$ find the temperature as Fahrenheit scale.

Solution

°F = 9/5**°C** + 32

°F = 9/5× 48**°C** + 32 = **118.4°F**

6. The sum of the ages of the father and son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?

Solution

Son's current age = x year

Father's current age = 72 − x year

Six years ago, Son's age = x − 6 year

Six years ago, Father's age = (72 – x) – 6 = 66 – x year

Six years ago, according to condition: $66 - x = 2(x - 6)$

Simplifying we get: $x = 26$

Six years ago, Son's age = 26 − 6 = **20 year**

7. Mirha sells a toy for Rs. 1520. What will the selling price be to get a 15% profit?

Solution

$\text{CP} = \text{Rs.}\,1520$

$\text{Profit\%} = 15\%$

$\text{Profit} = 15\% \text{ of } 1520 = \frac{15}{100} \times 1520 = \text{Rs.}\,228$

$\text{SP} = \text{CP} + \text{Profit}$

$\text{SP} = \text{Rs.}\,1520 + \text{Rs.}\,228$

$\mathbf{SP = Rs.\,1748}$

8. The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%?

Solution

Taxable Income = Total Income − Exempted Amount

Taxable Income = $\text{Rs.}\,960000 - \text{Rs.}\,130000$

Taxable Income = $\text{Rs.}\,830000$

Tax Rate = $0.75\% = 0.0075$

Tax Amount = Taxable Income × Tax Rate

Tax Amount = $\text{Rs.}\,830000 \times 0.0075$

Tax Amount = Rs. 6225

9. Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

Solution

Principal Amount (P) = $\text{Rs.}\,375000$

Rate of Interest (R) = $14\% = 0.14$

Time (T) = 1 year

Compound Interest (CI) = $\text{P} \times \text{R} \times \text{T}$

Compound Interest (CI) = $\text{Rs.}\,375000 \times 0.14 \times 1$

Compound Interest (CI) = Rs. 52500

2nd Method

Principal Amount (P) = $\text{Rs.}\,375000$

Rate of Interest (R) = $14\% = 0.14$

Time (T) = 1 year

Compound Interest (CI) = $\text{P} \times (1 + \text{R})^{\text{T}} - \text{P}$

Compound Interest (CI) = $\text{Rs.}\,375000 \times (1 + 0.14)^{1} - \text{Rs.}\,375000$

Compound Interest (CI) = Rs. 52500

REVIEW EXERCISE 1

1. Four options are given against each statement. Encircle the correct option.

(i) $\sqrt{7}$ is:

(a) integer (b) rational number

(c) ✓ irrational number (d) natural number

(ii) π and e are:

(a) natural numbers (b) integers

(c) rational numbers (d) ✓ irrational numbers

(iii) If n is not a perfect square, then $\sqrt{n}$ is:

(a) rational number (b) natural number

(c) integer (d) ✓ irrational number

(iv) $\sqrt{3}+\sqrt{5}$ is:

(a) whole number (b) integer

(c) rational number (d) ✓ irrational number

(v) For all $x \in R$, $x = x$ is called:

(a) ✓ reflexive property (b) transitive number

(c) symmetric property (d) trichotomy property

(vi) Let $a, b, c \in R$, then $a > b$ and $b > c \Rightarrow a > c$ is called __________ property.

(a) trichotomy (b) ✓ transitive

(c) additive (d) multiplicative

(vii) $2^x \times 8^x = 64$ then $x =$

(a) ✓ $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(viii) Let $a, b \in R$, then $a = b$ and $b = a$ is called __________ property.

(a) reflexive (b) ✓ symmetric

(c) transitive (d) additive

(ix) $\sqrt{75}+\sqrt{27}=$

(a) $\sqrt{102}$ (b) $9\sqrt{3}$ (c) $5\sqrt{3}$ (d) ✓ $8\sqrt{3}$

(x) The product of $(3+\sqrt{5})(3-\sqrt{5})$ is:

(a) prime number (b) odd number

(c) irrational number (d) ✓ rational number

2. If $a=\frac{3}{2}$, $b=\frac{5}{3}$ and $c=\frac{7}{5}$, then verify that

(i) $a(b+c)=ab+ac$ (ii) $(a+b)c=ac+bc$

Solution

i. $\boldsymbol{a(b+c)=ab+ac}$

$\text{L.H.S} = \text{a(b + c)} = \frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right) = \frac{3}{2}\left(\frac{25+21}{15}\right) = \frac{3}{2}\left(\frac{46}{15}\right) = \frac{138}{30} = \frac{23}{5}$

$\text{R.H.S} = ab+ac = \frac{3}{2}\left(\frac{5}{3}\right)+\frac{3}{2}\left(\frac{7}{5}\right) = \frac{15}{6}+\frac{21}{10} = \frac{5}{2}+\frac{21}{10} = \frac{46}{10} = \frac{23}{5}$

Hence $a(b+c)=ab+ac$

ii. $\boldsymbol{(a+b)c=ac+bc}$

$\text{L.H.S} = (a+b)c = \left(\frac{3}{2}+\frac{5}{3}\right)\frac{7}{5} = \left(\frac{9+10}{6}\right)\frac{7}{5} = \left(\frac{19}{6}\right)\frac{7}{5} = \frac{133}{30}$

$\text{R.H.S} = ac+bc = \left(\frac{3}{2}\right)\frac{7}{5}+\left(\frac{5}{3}\right)\frac{7}{5} = \frac{21}{10}+\frac{35}{15} = \frac{21}{10}+\frac{7}{3} = \frac{133}{30}$

Hence $(a+b)c=ac+bc$

3. If $a=\frac{4}{3}$, $b=\frac{5}{2}$, $c=\frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

Solution

We have to verify

$(a+b)+c=a+(b+c)$ and $(a\times b)\times c=a\times(b\times c)$

i. $\boldsymbol{(a+b)+c=a+(b+c)}$

$\text{L.H.S} = (a+b)+c = \left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4} = \left(\frac{8+15}{6}\right)+\frac{7}{4} = \frac{23}{6}+\frac{7}{4} = \frac{67}{12}$

$\text{R.H.S} = a+(b+c) = \frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right) = \frac{4}{3}+\left(\frac{10+7}{4}\right) = \frac{4}{3}+\frac{17}{4} = \frac{67}{12}$

Hence $(a+b)+c=a+(b+c)$

ii. $\boldsymbol{(a\times b)\times c=a\times(b\times c)}$

$\text{L.H.S} = (a\times b)\times c = \left(\frac{4}{3}\times\frac{5}{2}\right)\times\frac{7}{4} = \frac{20}{6}\times\frac{7}{4} = \frac{10}{3}\times\frac{7}{4} = \frac{70}{12} = \frac{35}{6}$

$\text{R.H.S} = a\times(b\times c) = \frac{4}{3}\times\left(\frac{5}{2}\times\frac{7}{4}\right) = \frac{4}{3}\times\frac{35}{8} = \frac{140}{24} = \frac{35}{6}$

Hence $(a\times b)\times c=a\times(b\times c)$

4. Is 0 a rational number? Explain.

Solution

Yes, zero is a rational number. A rational number is defined as a number that can be expressed as the ratio of two integers, i.e., $\frac{a}{b}$, where a and b are integers and b is non-zero. Zero can be expressed as a ratio of two integers, such as: 0 = 0/1

In this case, both 0 and 1 are integers, and 1 is non-zero. Therefore, zero meets the definition of a rational number.

5. State trichotomy property of real numbers.

Solution

For any two real numbers a and b, exactly one of the following is true:

1. a < b   2. a = b   3. a > b

6. Find two rational numbers between 4 and 5.

Solution

$q_1 = \frac{1}{2}(4+5) = \frac{9}{2}$ and $q_2 = \frac{1}{2}\left(\frac{9}{2}+5\right) = \frac{1}{2}\left(\frac{19}{2}\right) = \frac{19}{4}$

Hence required rational are $\mathbf{\frac{9}{2}, \frac{19}{4}}$

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$ (ii) $\sqrt[3]{(27)^{2x}}$ (iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

Solution

i. $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \left(\frac{x^{15}y^{35}}{z^{20}}\right)^{\frac{1}{5}} = \frac{x^{15\times\frac{1}{5}}y^{35\times\frac{1}{5}}}{z^{20\times\frac{1}{5}}} = \mathbf{\frac{x^3y^7}{z^4}}$

ii. $\sqrt[3]{(27)^{2x}} = (27)^{\frac{2x}{3}} = (3^3)^{\frac{2x}{3}} = \mathbf{3^{2x}}$

iii. $\frac{6(3)^{n+2}}{(3)^{n+1}-3^n} = \frac{3^n(6\times3^2)}{3^n(3-1)} = \frac{6\times9}{2} = \mathbf{27}$

8. The sum of three consecutive odd integers is 51. Find the three integers.

Solution

Let the three consecutive odd integers be x, x+2, and x+4.

$x + (x+2) + (x+4) = 51$

$3x + 6 = 51$

$3x = 45$

$x = 15$

Now that we know x, we can find the other two integers:

$x+2 = 17$

$x+4 = 19$

So, the three consecutive integers are **15**, **17**, and **19**.

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Solution

Let's say the number of balls in the smaller bucket is x. Since the other bucket has 28 more balls, the number of balls in the larger bucket is x + 28.

We know that the total number of balls is 96, so we can set up the equation:

$x + (x + 28) = 96$

$2x + 28 = 96$

$2x = 68$

$x = 34$

So, the smaller bucket has 34 balls.

The larger bucket has 34 + 28 = 62 balls.

Therefore, the two buckets have **34** and **62** balls, respectively.

10. Salma invested Rs. 3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

Solution

Initial Investment = Rs. 3,50,000

Rate of interest for the first 2 years = $7\frac{1}{4}\%$ = 7.25% per annum

Interest for the first 2 years = (3,50,000 × 7.25% × 2) = Rs. 50,750

Rate of interest for the next 5 years = 8% per annum

Interest for the next 5 years = (3,50,000 × 8% × 5) = Rs. 1,40,000

Amount after 7 years = 3,50,000 + 50,750+ 1,40,000 = Rs. 5,40,750

Therefore, Salma had **Rs. 5,40,750** at the end of 7 years.