Unit 5 Linear Equations and Inequalities

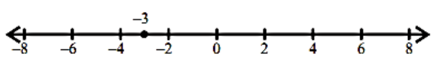
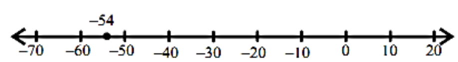
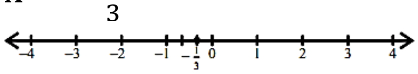
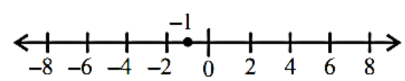
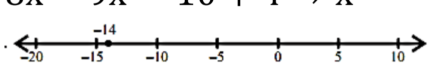
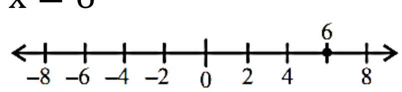
EXERCISE 5.1

1. Solve and represent the solution on a real line.

(i) $12x + 30 = -6$ (ii) $\frac{x}{3} + 6 = -12$ (iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

(iv) $2 = 7(2x + 4) + 12x$ (v) $\frac{2x-1}{3} - \frac{3}{4}\frac{x}{} = \frac{5}{6}$ (vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$

Solution

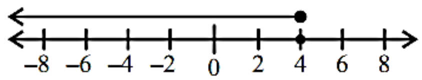
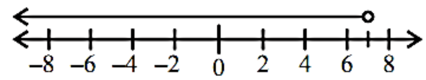
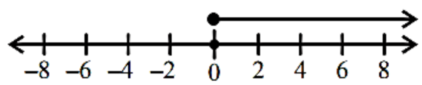
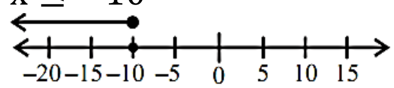
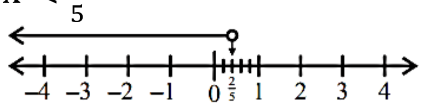
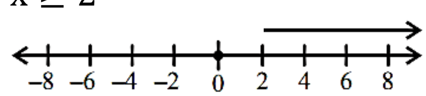
(i) $12x + 30 = -6$ $12x = -6 - 30$ $12x = -36$ $x = -\frac{36}{12}$ $x = -3$ 	**(ii)** $\frac{x}{3} + 6 = -12$ $\frac{x}{3} = -12 - 6$ $\frac{x}{3} = -18$ $x = -18 \times 3 \Rightarrow x = -54$ 
(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$ $12 \times \left(\frac{x}{2}\right) - 12 \times \left(\frac{3x}{4}\right) = 12 \times \left(\frac{1}{12}\right)$ $6x - 9x = 1 \Rightarrow -3x = 1$ $x = -\frac{1}{3}$ 	**(iv)** $2 = 7(2x + 4) + 12x$ $2 = 14x + 28 + 12x$ $2 - 28 = 14x + 12x$ $-26 = 26x \Rightarrow x = -\frac{26}{26}$ $x = -1$ 
(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$ $12 \times \left(\frac{2x-1}{3}\right) - 12 \times \left(\frac{3x}{4}\right) = 12 \times \left(\frac{5}{6}\right)$ $4(2x - 1) - 9x = 10$ $8x - 4 - 9x = 10$ $8x - 9x = 10 + 4 \Rightarrow x = -14$ 	**(vi)** $-\frac{5x}{10} = 9 - \frac{10}{5}x$ $10 \times \left(-\frac{5x}{10}\right) = 10 \times (9) - 10 \times \left(\frac{10}{5}x\right)$ $-5x = 90 - 20x$ $-5x + 20x = 90 \Rightarrow 15x = 90$ $x = 6$ 

2. Solve each inequality and represent the solution on a real line.

(i) $x-6\le -2$ (ii) $-9>-16+x$ (iii) $3+2x\ge 3$

(iv) $6(x+10)\ \le 0$ (v) $\frac{5}{3}x-\frac{3}{4}<\frac{-1}{12}$ (vi) $\frac{1}{4}x-\frac{1}{2}\le -1+\frac{1}{2}x$

Solution

(i) $x-6\le -2$ $x\le -2+6$ $x\le 4$ 	**(ii)** $-9>-16+x$ $-9+16>x$ $7>x$ or $x<7$ 
(iii) $3+2x\ge 3$ $2x\ge 3-3$ $2x\ge 0$ $x\ge 0$ 	**(iv)** $6(x+10)\le 0$ $6x+60\le 0$ $6x\le -60$ $x\le -\frac{60}{6}$ $x\le -10$ 
(v) $\frac{5}{3}x-\frac{3}{4}<-\frac{1}{12}$ $12\times\left(\frac{5}{3}x\right)-12\times\left(\frac{3}{4}\right)<12\times\left(-\frac{1}{12}\right)$ $4(5x)-9<-1$ $20x<-1+9$ $20x<8$ $x<\frac{8}{20}$ $x<\frac{2}{5}$ 	**(vi)** $\frac{1}{4}x-\frac{1}{2}\le -1+\frac{1}{2}x$ $4\times\left(\frac{1}{4}x\right)-4\times\left(\frac{1}{2}\right)\le 4\times(-1)+4\times\left(\frac{1}{2}x\right)$ $x-2\le -4+2x$ $-2+4\le 2x-x$ $2\le x$ $x\ge 2$ 

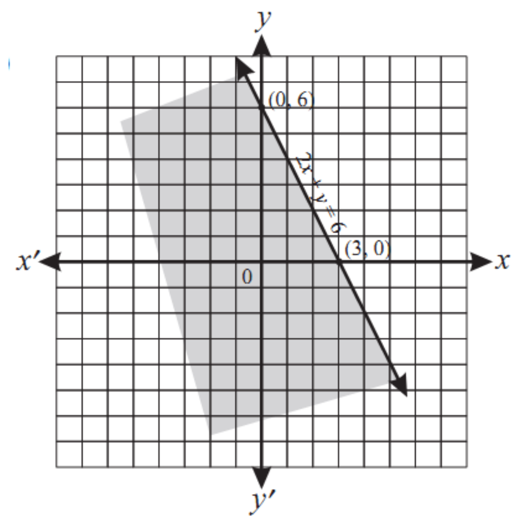
3. Shade the solution region for the following linear inequalities in xy-plane:

(i) $2x+y\le 6$ (ii) $3x+7y\ge 21$ (iii) $3x-2y\ge 6$

(iv) $5x-4y\le 20$ (v) $2x+1\ge 0$ (vi) $3y-4\le 0$

Solution

3 (i) $2x + y \le 6$



Associated equations: $2x + y = 6$

To find Points:

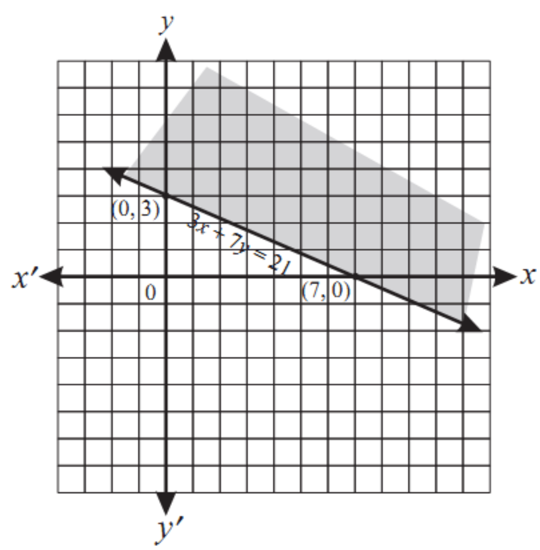
Put $x = 0$, $y = 6$ then point is $(0,6)$

Put $y = 0$, $x = 3$ then point is $(3,0)$

To check Region put $(0, 0)$ in given eq.

$0 < 6$ true, graph towards the origin

3 (ii) $3x + 7y \ge 21$



Associated equations: $3x + 7y = 21$

To find Points

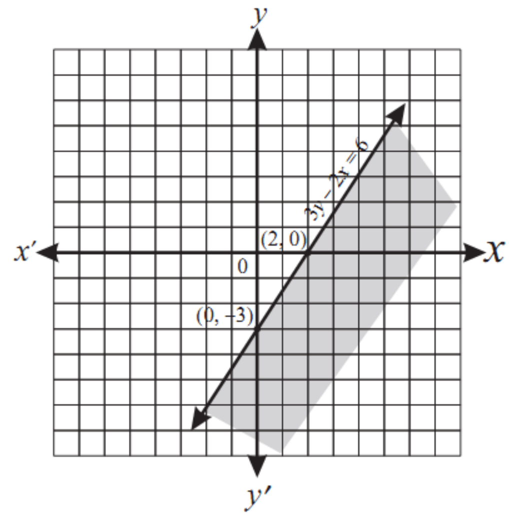
Put $x = 0$, $y = 3$ then point is $(0,3)$

Put $y = 0$, $x = 7$ then point is $(7,0)$

To check Region put $(0, 0)$ in given eq.

$0 > 21$ false, graph away from origin

3 (iii) $\mathbf{3x - 2y \geq 6}$



Associated equations: $3x - 2y = 6$

To find Points:

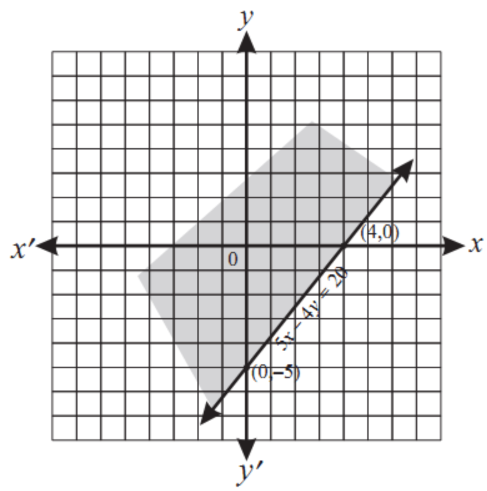
Put $\text{x} = 0$, $\text{y} = -3$ then point is $(0, -3)$

Put $\text{y} = 0$, $\text{x} = 2$ then point is $(2,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in given eq.

$0 > 6$ false, graph away from origin

3 (iv) $\mathbf{5x - 4y \leq 20}$



Associated equations: $5x - 4y = 20$

To find Points

Put $\text{x} = 0$, $\text{y} = -5$ then point is $(0, -5)$

Put $\text{y} = 0$, $\text{x} = 4$ then point is $(4,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in given eq.

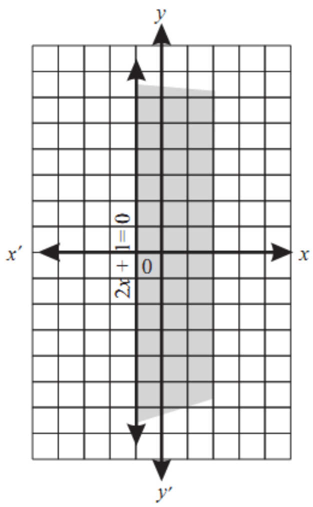
$0 < 20$ true, graph towards the origin

3 (v) $2x + 1 \geq 0$

Associated equations: $2x + 1 = 0$

Point: $x = -\frac{1}{2}$

Region: $1 > 0$ true, graph towards the origin

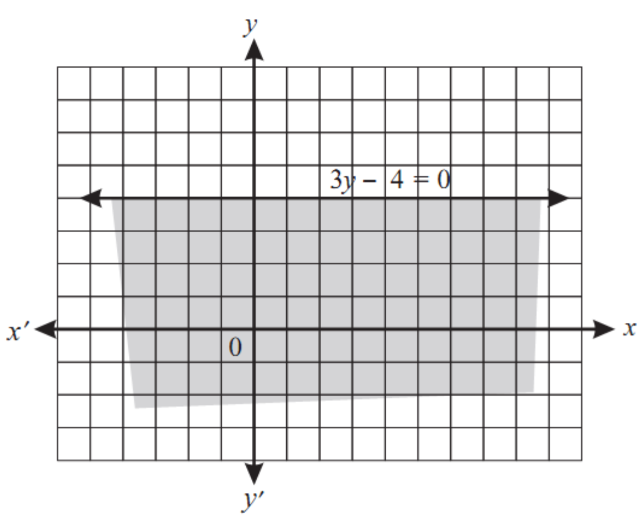


3 (vi) $3y - 4 \leq 0$

Associated equations: $3y - 4 = 0$

Point: $y = \frac{4}{3}$

Region: $0 < 4$ true, graph towards the origin



4. Indicate the solution region of the following linear inequalities by shading:

(i) $2x-3y\leq 6$, $2x+3y\leq 12$ (ii) $x+y\geq 5$, $-y+x\leq 1$ (iii) $3x+7y\geq 21$, $x-y\leq 2$

(iv) $4x-3y\leq 12$, $x\geq -\frac{3}{2}$ (v) $3x+7y\geq 21$, $y\leq 4$ (vi) $5x+7y\leq 35$, $x-2y\leq 2$

Solution

4 (i)

$\mathbf{2x-3y\leq 6}$ ………(i)

$\mathbf{2x+3y\leq 12}$ ………(ii)

Associated equations

$2x-3y=6$ ………(iii)

$2x+3y=12$ ………(iv)

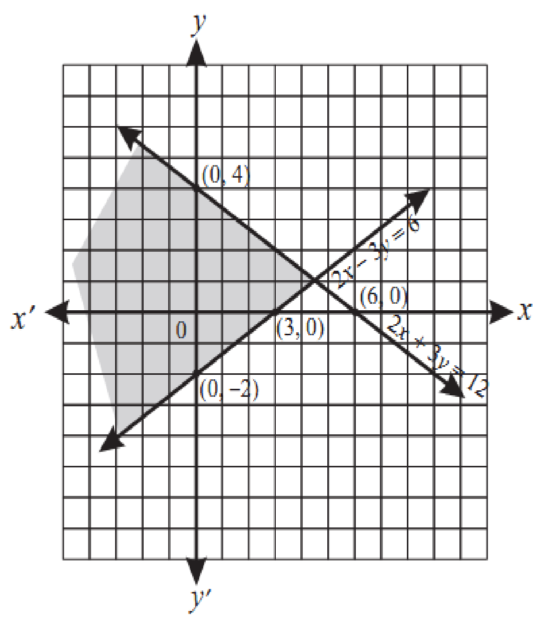
To find Points

(iii) $\Rightarrow$ Put $x=0$, $y=-2$ then point is $(0,-2)$

(iii) $\Rightarrow$ Put $y=0$, $x=3$ then point is $(3,0)$

(iv) $\Rightarrow$ Put $x=0$, $y=4$ then point is $(0,4)$

(iv) $\Rightarrow$ Put $y=0$, $x=6$ then point is $(6,0)$



To check Region put $(0,0)$ in (i) and (ii)

(i) $\Rightarrow 0<6$ true, graph towards the origin

(ii) $\Rightarrow 0<12$ true, graph towards the origin

4 (ii)

$\mathbf{x + y \geq 5}$ ………$(\mathbf{i})$

$\mathbf{-y + x \leq 1}$ ………$(\mathbf{ii})$

Associated equations

$x + y = 5$ ………(iii)

$x - y = 1$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 5$ then point is $(0,5)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 5$ then point is $(5,0)$

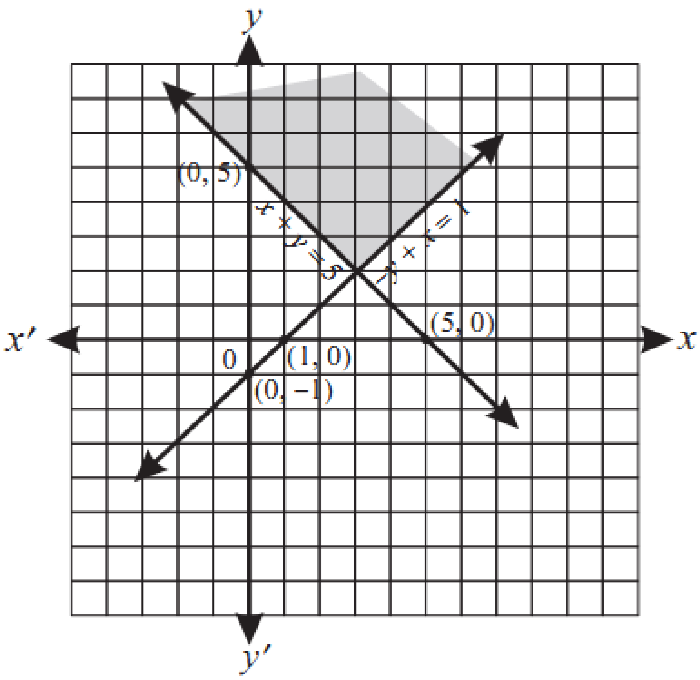
(iv) $\Rightarrow$ Put $x = 0$, $y = -1$ then point is $(0,-1)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 1$ then point is $(1,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 > 5$ false, graph away from origin

(ii) $\Rightarrow 0 < 1$ true, graph towards the origin



4 (iii)

$\mathbf{3x + 7y \geq 21}$ $\mathbf{\ldots\ldots\ldots(i)}$

$\mathbf{x - y \leq 2}$ $\mathbf{\ldots\ldots\ldots(ii)}$

Associated equations

$3x + 7y = 21$ $\ldots\ldots\ldots(iii)$

$x - y = 2$ $\ldots\ldots\ldots(iv)$

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 3$ then point is $(0,3)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 7$ then point is $(7,0)$

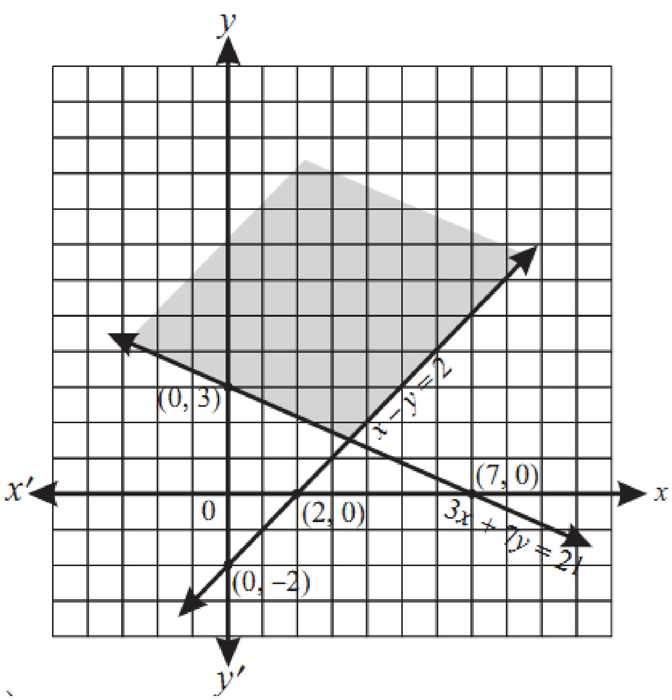
(iv) $\Rightarrow$ Put $x = 0$, $y = -2$ then point is $(0,-2)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 2$ then point is $(2,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 > 21$ false, graph away from origin

(ii) $\Rightarrow 0 < 2$ true, graph towards the origin



4 (iv)

$\mathbf{4x - 3y \leq 12}$ ………(**i**)

$\mathbf{x \geq -\frac{3}{2}}$ ………(**ii**)

Associated equations

$4x - 3y = 12$ ………(iii)

$x = -\frac{3}{2}$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = -4$ then point is $(0, -4)$

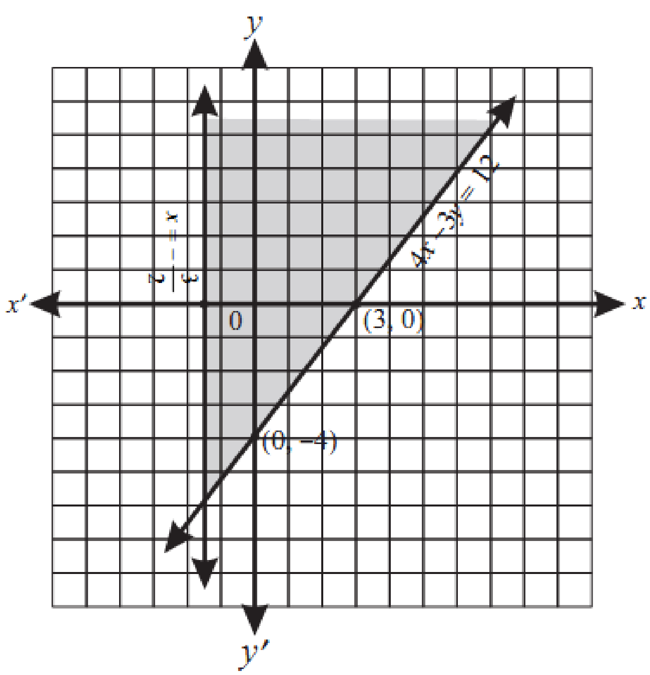
(iii) $\Rightarrow$ Put $y = 0$, $x = 3$ then point is $(3,0)$

(iv) $\Rightarrow$ we have $y = 0$, $x = -\frac{3}{2}$ then point is $\left(-\frac{3}{2}, 0\right)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 12$ true, graph towards the origin

(ii) $\Rightarrow 0 > -\frac{3}{2}$ true, graph towards the origin



4 (v)

$\mathbf{3x + 7y \geq 12}$ **………(i)**

$\mathbf{y \leq 4}$ **………(ii)**

Associated equations

$3x + 7y = 12$ ………(iii)

$y = 4$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 3$ then point is $(0,3)$

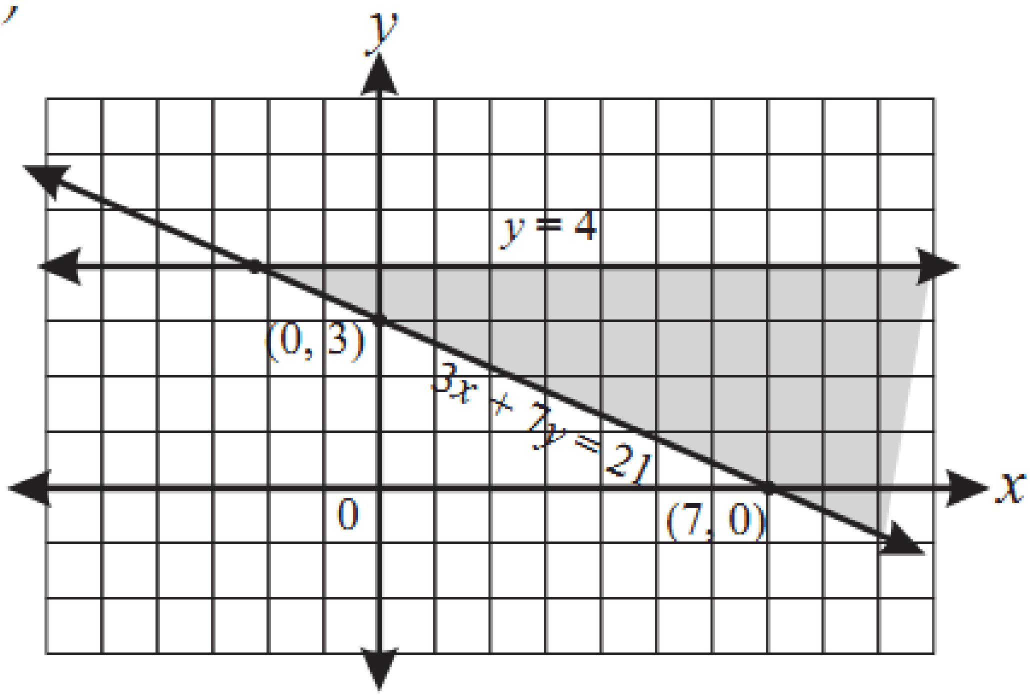
(iii) $\Rightarrow$ Put $y = 0$, $x = 7$ then point is $(7,0)$

(iv) $\Rightarrow$ we have $x = 0$, $y = 4$ then point is $(0,4)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 > 12$ false, graph away from origin

(ii) $\Rightarrow 0 < 4$ true, graph towards the origin



4 (vi)

$\mathbf{5x + 7y \leq 35}$ ....(i)

$\mathbf{x - 2y \leq 2}$ ....(ii)

Associated equations

$5x + 7y = 35$ ....(iii)

$x - 2y = 2$ ....(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 5$ then point is $(0,5)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 7$ then point is $(7,0)$

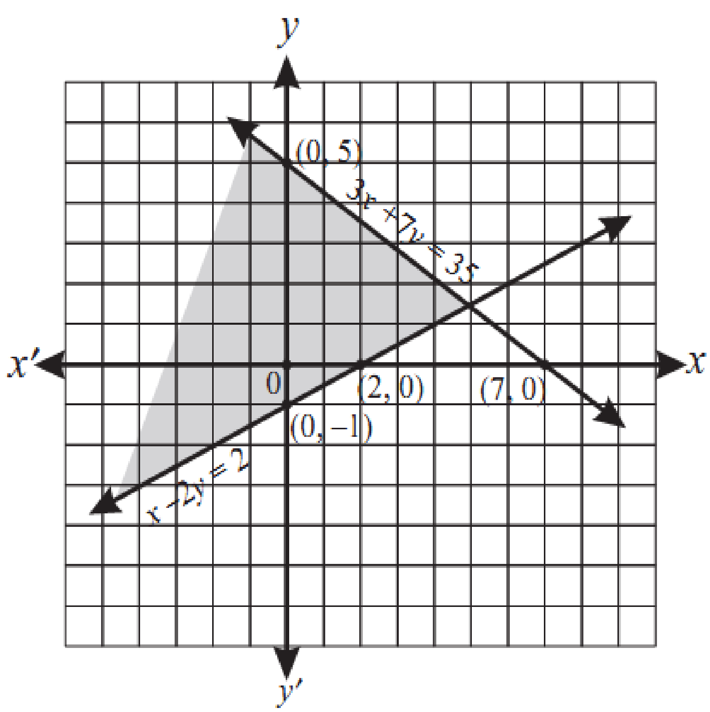
(iv) $\Rightarrow$ Put $x = 0$, $y = -1$ then point is $(0,-1)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 2$ then point is $(2,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 35$ true, graph towards the origin

(ii) $\Rightarrow 0 < 2$ true, graph towards the origin



EXERCISE 5.2

1. Maximize $f(x, y) = 2x + 5y$; subject to the constraints

$$2y - x \leq 8 \quad ; \quad x - y \leq 4 \quad ; \quad x \geq 0; \quad y \geq 0$$

Solution

$-x + 2y \leq 8$ ....(i)

$x - y \leq 4$ ....(ii)

Associated equations

$-x + 2y = 8$ ....(iii)

$x - y = 4$ ....(iv)

To find Points

(iii) ⇒ Put $x = 0$, $y = 4$ then point is $(0,4)$

(iii) ⇒ Put $y = 0$, $x = -8$ then point is $(-8,0)$

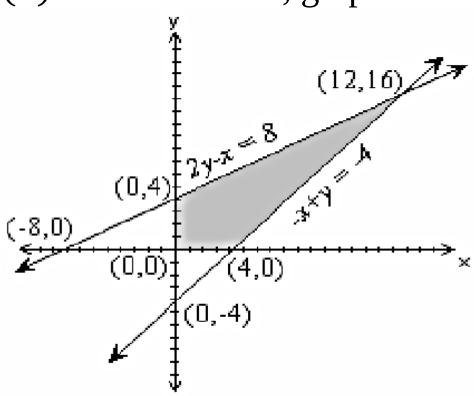
(iv) ⇒ Put $x = 0$, $y = -4$ then point is $(0,-4)$

(iv) ⇒ Put $y = 0$, $x = 4$ then point is $(4,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) ⇒ $0 < 8$ true, graph towards the origin

(ii) ⇒ $0 < 4$ true, graph towards the origin



Solve $(\mathbf{iii}) + (\mathbf{iv})$

$(-x + 2y) + (x - y) = 8 + 4$ we have $y = 12$

Put $y = 12$ in (iii) we have $x = 16$ and $D(16,12)$

Corner Points of Feasible Region: $A(0,0), B(4,0), C(0,4), D(16,12)$

At A: $z = f(0,0) = 2(0) + 5(0) = 0$

At B: $z = f(4,0) = 2(4) + 5(0) = 8$

At C: $z = f(0,4) = 2(0) + 5(4) = 20$

At D: $z = f(16,12) = 2(16) + 5(12) = 92$

So $z = 2x + 5y$ is maximum at $(16,12)$

2. Maximize $f(x, y) = x + 3y$; subject to the constraints

$$2x + 5y \leq 30 \quad ; \quad 5x + 4y \leq 20 \quad ; \quad x \geq 0 \; ; y \geq 0$$

Solution

$2x + 5y \leq 30$ ………(i)

$5x + 4y \leq 20$ ………(ii)

Associated equations

$2x + 5y = 30$ ………(iii)

$5x + 4y = 20$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 6$ then point is $(0,6)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 15$ then point is $(15,0)$

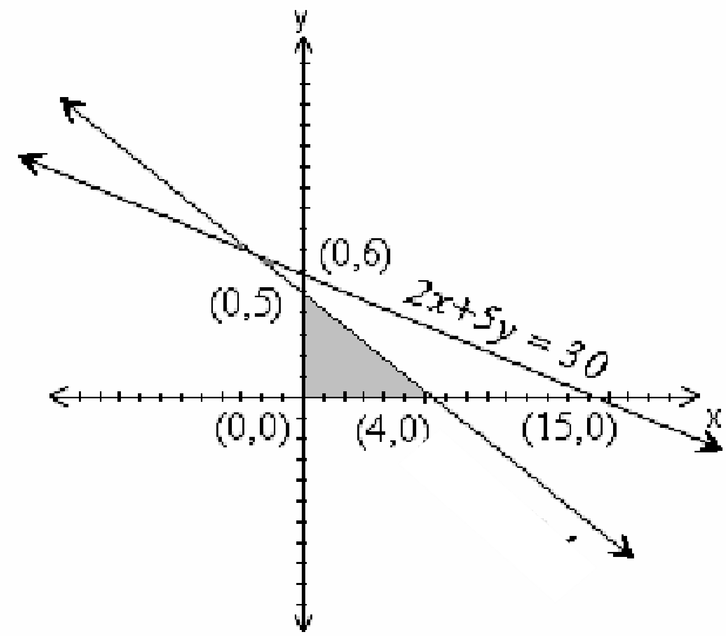
(iv) $\Rightarrow$ Put $x = 0$, $y = 5$ then point is $(0,5)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 4$ then point is $(4,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 30$ true, graph towards the origin

(ii) $\Rightarrow 0 < 20$ true, graph towards the origin



Corner Points of Feasible Region: $A(0,0), B(4,0), C(0,5)$

At A: $z = f(0,0) = (0) + 3(0) = 0$

At B: $z = f(4,0) = (4) + 3(0) = 4$

At C: $z = f(0,5) = (0) + 3(5) = 15$

So $z = x + 3y$ is maximum at $(0,5)$

3. Maximize $z = 2x + 3y$; subject to the constraints:

$$2x + y \leq 4 \quad ; \quad 4x - y \leq 4 \quad ; \quad x \geq 0: \ \ y \geq 0$$

Solution

$2x + y \leq 4$ ………(i)

$4x - y \leq 4$ ………(ii)

Associated equations

$2x + y = 4$ ………(iii)

$4x - y = 4$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 4$ then point is $(0,4)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 2$ then point is $(2,0)$

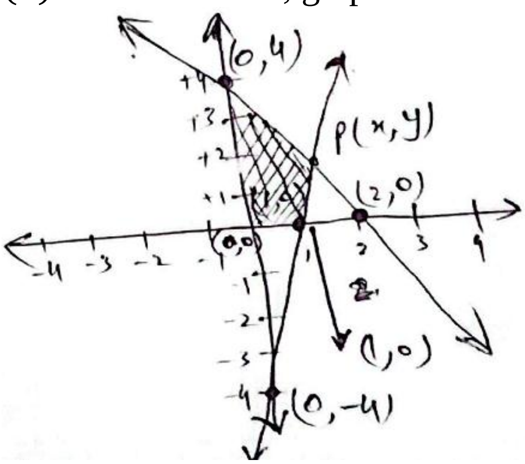
(iv) $\Rightarrow$ Put $x = 0$, $y = -4$ then point is $(0,-4)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 1$ then point is $(1,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 4$ true, graph towards the origin

(ii) $\Rightarrow 0 < 4$ true, graph towards the origin



Solve $(\mathbf{iii}) + (\mathbf{iv})$

$(2x + y) + (4x - y) = 4 + 4$ we have $x = \frac{4}{3}$

Put $x = \frac{4}{3}$ in (iii) we have $y = \frac{4}{3}$ and the intersecting point is $\left(\frac{4}{3}, \frac{4}{3}\right)$

Corner Points: $A(0,0), B(1,0), C(0,4), P\left(\frac{4}{3}, \frac{4}{3}\right)$

At A: $z = f(0,0) = 2(0) + 3(0) = 0$

At B: $z = f(1,0) = 2(1) + 3(0) = 2$

At C: $z = f(0,4) = 2(0) + 3(4) = 12$

At P: $z = f\left(\frac{4}{3}, \frac{4}{3}\right) = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right) = 6.66$

So $z = 2x + 3y$ is maximum at $(0,4)$

4. Minimize $z = 2x + y$; subject to the constraints:

$$x + y \geq 3 \quad ; \quad 7x + 5y \leq 35 \quad ; \quad x \geq 0; \quad y \geq 0$$

Solution

$x + y \geq 3$ ....(i)

$7x + 5y \leq 35$ ....(ii)

Associated equations

$x + y = 3$ ....(iii)

$7x + 5y = 35$ ....(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 3$ then point is $(0,3)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 3$ then point is $(3,0)$

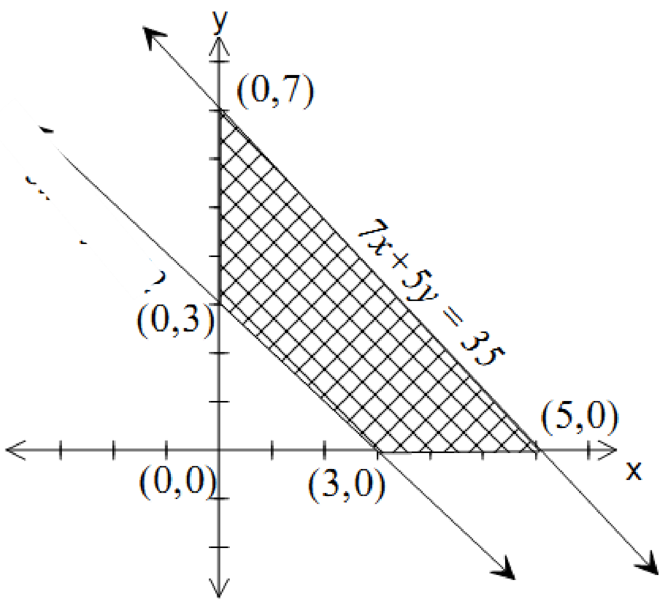
(iv) $\Rightarrow$ Put $x = 0$, $y = 7$ then point is $(0,7)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 5$ then point is $(5,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 > 3$ false, graph away from the origin

(ii) $\Rightarrow 0 < 35$ true, graph towards the origin



Corner Points: $A(3,0), B(0,3), C(5,0), P(0,7)$

At A: $z = f(3,0) = 2(3) + (0) = 6$

At B: $z = f(0,3) = 2(0) + (3) = 3$

At C: $z = f(5,0) = 2(5) + (0) = 10$

At P: $z = f(0,7) = 2(0) + (7) = 7$

So $z = 2x + y$ is minimum at $(0,3)$

5. Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints:

$$2x + y \le 8 \quad ; \quad x + 2y \le 14 \quad ; \quad x \ge 0; \quad y \ge 0$$

Solution

$2x + y \le 8$ ....(i)

$x + 2y \le 14$ ....(ii)

Associated equations

$2x + y = 8$ ....(iii)

$x + 2y = 14$ ....(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 8$ then point is $(0,8)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 4$ then point is $(4,0)$

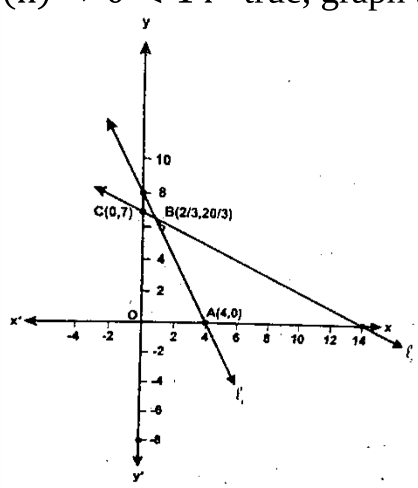
(iv) $\Rightarrow$ Put $x = 0$, $y = 7$ then point is $(0,7)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 14$ then point is $(14,0)$

To check Region put $(0,0)$ in (i) and (ii)

(i) $\Rightarrow 0 < 8$ true, graph towards the origin

(ii) $\Rightarrow 0 < 14$ true, graph towards the origin



Solve $2(iii) - (iv)$

$(4x + 2y) - (x + 2y) = 16 - 14$ we have $x = \frac{2}{3}$

Put $x = \frac{2}{3}$ in (iii) we have $y = \frac{20}{3}$ and $C\left(\frac{2}{3}, \frac{20}{3}\right)$

Corner Points: $A(0,0), B(4,0), C\left(\frac{2}{3}, \frac{20}{3}\right), D(0,7)$

At A: $z = f(0,0) = 2(0) + 3(0) = 0$

At B: $z = f(4,0) = 2(4) + 3(0) = 8$

At C: $z = f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = 21.33$

At D: $z = f(0,7) = 2(0) + 3(7) = 21$

So $z = 2x + 3y$ is maximum at $\left(\frac{2}{3}, \frac{20}{3}\right)$

6. Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$$3x + 5y \geq 15 \quad ; \quad x + 6y \geq 9 \quad ; \quad x \geq 0; \quad y \geq 0$$

Solution

$3x + 5y \geq 15$ ....(i)

$x + 6y \geq 9$ ....(ii)

Associated equations

$3x + 5y = 15$ ....(iii)

$x + 6y = 9$ ....(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 3$ then point is $(0,3)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 5$ then point is $(5,0)$

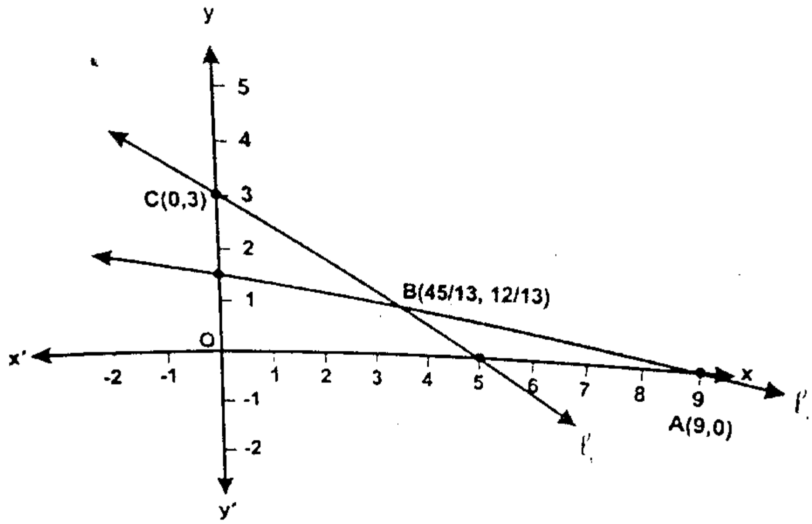
(iv) $\Rightarrow$ Put $x = 0$, $y = \frac{3}{2}$ then point is $\left(0, \frac{3}{2}\right)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 9$ then point is $(9,0)$

To check Region put $(0,0)$ in (i) and (ii)

(i) $\Rightarrow 0 > 15$ false, graph away from the origin

(ii) $\Rightarrow 0 > 9$ false, graph away from the origin



Solve $3(\text{iv}) - (\text{iii})$

$(3x + 18y) - (3x + 5y) = 27 - 15$ we have $y = \frac{12}{13}$

Put $y = \frac{12}{13}$ in (iii) we have $x = \frac{45}{13}$ and $B\left(\frac{45}{13}, \frac{12}{13}\right)$

Corner Points: $A(0,3), B\left(\frac{45}{13}, \frac{12}{13}\right), C(9,0)$

At A: $z = f(0,3) = 3(0) + 3 = 3$

At B: $z = f\left(\frac{45}{13}, \frac{12}{13}\right) = 3\left(\frac{45}{13}\right) + \frac{12}{13} = 11.3$

At C: $z = f(9,0) = 3(9) + 0 = 27$

So $z = 3x + y$ is minimum at $(0,3)$ and maximum at $(9,0)$

REVIEW EXERCISE 5

1. Four options are given against each statement. Encircle the correct one.

i. In the following, linear equation is:

(a) $5x > 7$ (b) $4x - 2 < 1$
✓(c) $2x + 1 = 1$ (d) $4 = 1 + 3$

ii. Solution of $5x - 10 = 10$ is:

(a) 0 (b) 50
✓(c) 4 (d) -4

iii. If $7x + 4 < 6x + 6$, then x belongs to the interval

(a) $(2, \infty)$ (b) $[2,\infty)$
✓(c) $(-\infty, 2)$ (d) $(-\infty, 2]$

iv. A vertical line divides the plane into

(a) left half plane (b) right half plane
(c) full plane ✓(d) two half planes

v. The linear equation formed out of the linear inequality is called

(a) linear equation ✓(b) associated equation
(c) quadratic equal (d) none of these

vi. $3x + 4 < 0$ is:

(a) equation ✓(b) inequality
(c) not inequality (d) identity

vii. Corner point is also called:

(a) code ✓(b) vertex
(c) curve (d) region

viii. (0,0) is solution of inequality:

(a) $4x+5y>8$ (b) $3x+y>6$

(c) $-2x+3y<0$ (d) $x+y>4$

ix. The solution region restricted to the first quadrant is called:

(a) objective region (b) feasible region

(c) solution region (d) constraints region

x. A function that is to be maximized or minimized is called:

(a) solution function (b) objective function

(c) feasible function (d) none of these

2. Solve and represent their solutions on real line.

(i) $\dfrac{x+5}{3}=1-x$ (ii) $\dfrac{2x+1}{3}+\dfrac{1}{2}=1-\dfrac{x-1}{3}$

(iii) $3x+7<16$ (iv) $5(x-3)\geq 26x-(10x+4)$

Solution

(i) $\frac{x+5}{3}=1-x$

$x+5=3-3x$

$x+3x=3-5$

$4x=-2$

$x=-\frac{2}{4}$

$x=-\frac{1}{2}$

(ii) $\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$

$6\times\left(\frac{2x+1}{3}\right)+6\times\left(\frac{1}{2}\right)=6\times(1)-6\times\left(\frac{x-1}{3}\right)$

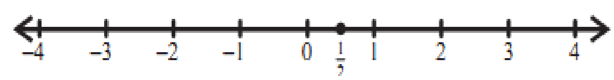
$2(2x+1)+3=6-2(x-1)$

$4x+2+3=6-2x+2$

$4x+2x=6+2-2-3$

$6x=3$

$x=\frac{1}{2}$



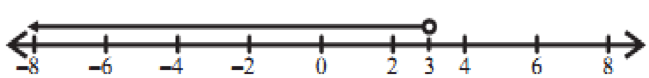
(iii) $3x+7<16$

$3x<16-7$

$3x<9$

$x<\frac{9}{3}$

$x<3$



(iv) $5(x-3)\geq 26x-(10x+4)$

$5x-15\geq 26x-10x-4$

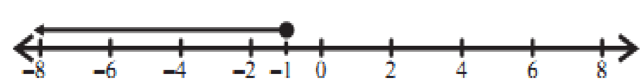
$5x-15\geq 16x-4$

$5x-16x\geq -4+15$

$-11x\geq 11$

$x\leq -\frac{11}{11}$

$x\leq -1$



3. Find the solution region of the following linear equalities:

(i) $3x - 4y \leq 12$; $3x + 2y \geq 3$

(ii) $2x + y \leq 4$; $x + 2y \leq 6$

Solution

3 (i)

$3x - 4y \leq 12$ ………(i)

$3x + 2y \geq 3$ ………(ii)

Associated equations

$3x - 4y = 12$ ………(iii)

$3x + 2y = 3$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = -3$ then point is $(0, -3)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 4$ then point is $(4,0)$

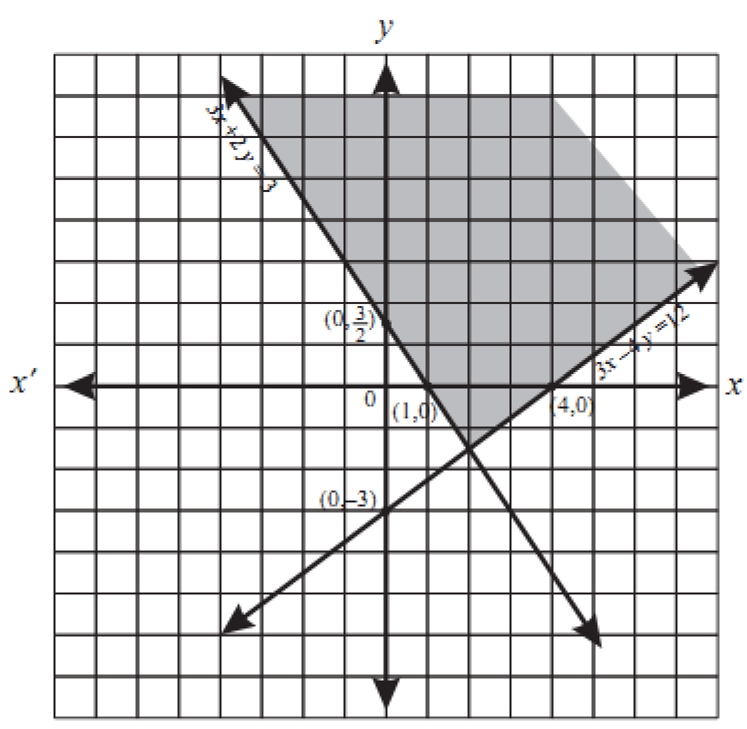
(iv) $\Rightarrow$ Put $x = 0$, $y = \frac{3}{2}$ then point is $\left(0, \frac{3}{2}\right)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 1$ then point is $(1,0)$

To check Region put $(\mathbf{0}, \mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 12$ true, graph towards the origin

(ii) $\Rightarrow 0 > 3$ false, graph away from the origin



3 (ii)

$2x + y \leq 4$ ....(i)

$x + 2y \leq 6$ ....(ii)

Associated equations

$2x + y = 4$ ....(iii)

$x + 2y = 6$ ....(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 4$ then point is $(0,4)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 2$ then point is $(2,0)$

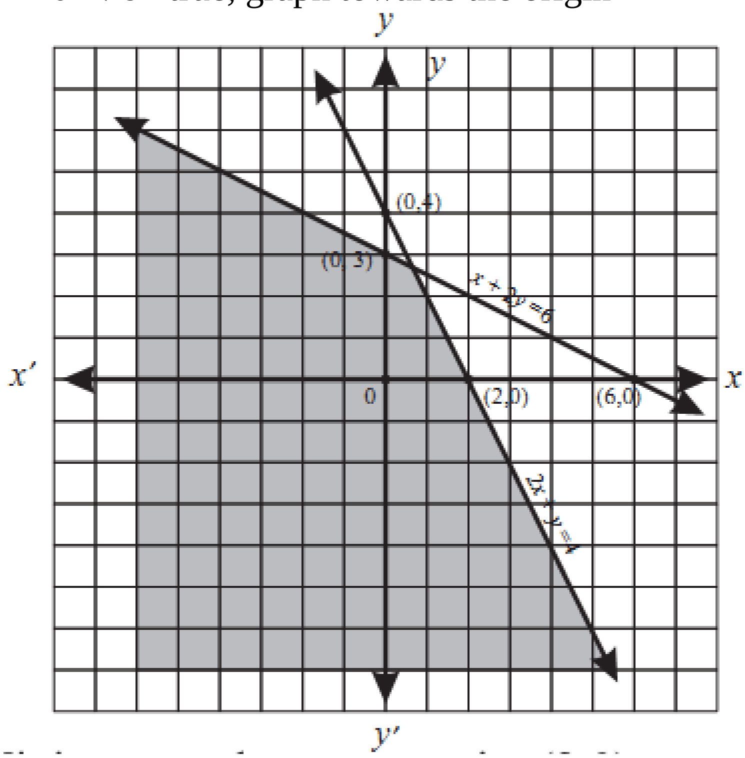
(iv) $\Rightarrow$ Put $x = 0$, $y = 3$ then point is $(0,3)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 6$ then point is $(6,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 < 4$ true, graph towards the origin

(ii) $\Rightarrow 0 < 6$ true, graph towards the origin



4. Find the maximum value of $g(x,y) = x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$ and $y \geq 0$.

Solution

$x + y \leq 4$

Associated equations

$x + y = 4$

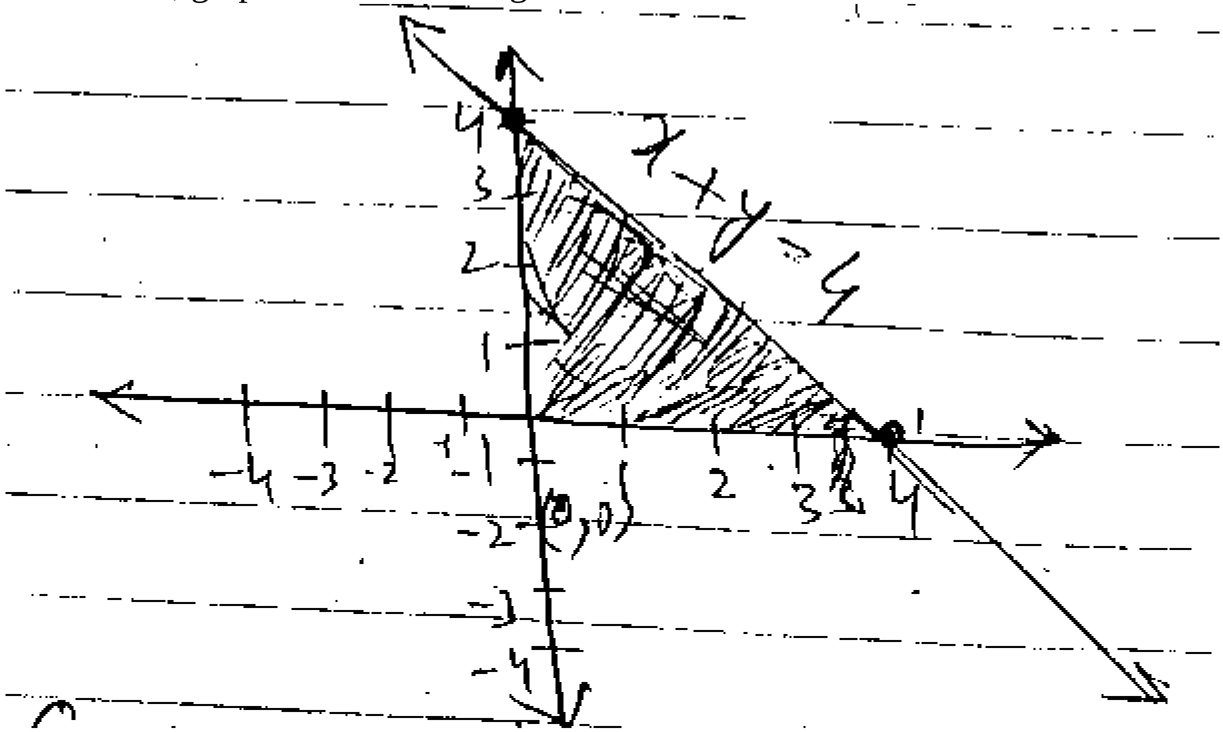
To find Points

$\Rightarrow$ Put $x = 0$, $y = 4$ then point is $(0,4)$

$\Rightarrow$ Put $y = 0$, $x = 4$ then point is $(4,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

$0 < 4$ true, graph towards the origin



Corner Points: $\mathrm{A}(0,0), \mathrm{B}(0,4), C(4,0)$

At A: $z = g(0,0) = (0) + 4(0) = 0$

At B: $z = g(0,4) = (0) + 4(4) = 16$

At C: $z = g(4,0) = (4) + 0(0) = 4$

So $z = x + 4y$ is maximum at $(0,4)$

5. Find the minimum value of $f(x,y) = 3x + 5y$ subject to constraints

$x + 3y \geq 3, \quad x + y \geq 2, \quad x \geq 0, \quad y \geq 0.$

Solution

$x + 3y \geq 3$ ………(i)

$x + y \geq 2$ ………(ii)

Associated equations

$x + 3y = 3$ ………(iii)

$x + y = 2$ ………(iv)

To find Points

(iii) $\Rightarrow$ Put $x = 0$, $y = 1$ then point is $(0,1)$

(iii) $\Rightarrow$ Put $y = 0$, $x = 3$ then point is $(3,0)$

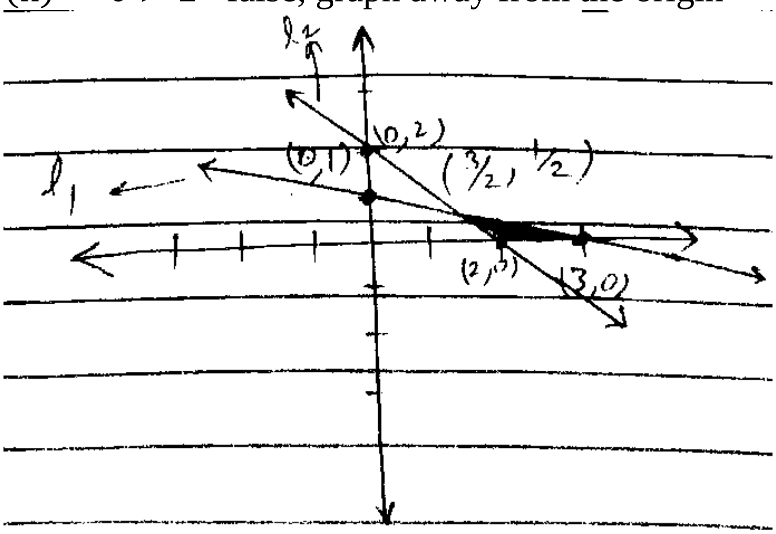
(iv) $\Rightarrow$ Put $x = 0$, $y = 2$ then point is $(0,2)$

(iv) $\Rightarrow$ Put $y = 0$, $x = 2$ then point is $(2,0)$

To check Region put $(\mathbf{0},\mathbf{0})$ in $(\mathbf{i})$ and $(\mathbf{ii})$

(i) $\Rightarrow 0 > 3$ false, graph away from the origin

(ii) $\Rightarrow 0 > 2$ false, graph away from the origin



Solve $(\mathbf{iii}) - (\mathbf{iv})$

$(x + 3y) - (x + y) = 3 - 2$ we have $y = \frac{1}{2}$

Put $y = \frac{1}{2}$ in (iii) we have $x = \frac{3}{2}$ and $P\left(\frac{3}{2}, \frac{1}{2}\right)$

Corner Points: $A(2,0), B(3,0), P\left(\frac{3}{2}, \frac{1}{2}\right)$

At A: $z = f(2,0) = 3(2) + 5(0) = 6$

At B: $z = f(3,0) = 3(3) + 5(0) = 9$

At P: $z = f\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = 8$

So $z = 3x + 5y$ is minimum at $(2,0)$ and maximum at $(3,0)$