Unit 6 Trigonometry

EXERCISE 6.1

1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:
   (i) 65° (ii) 135° (iii) –40° (iv) 210° (v) –150°

Solution

(i) 1^{st} (ii) 2^{nd} (iii) 4^{th} (iv) 3^{rd} (v) 3^{rd}

2. Convert the following into degrees, minutes, and seconds:
   (i) 123.456° (ii) 58.7891° (iii) 90.5678°

Solution

2(i): $\mathbf{123.456°}$

123

$0.456 \times 60 = 27.36$

$0.36 \times 60 = 21.6$

$123.456° \approx 123°\ 27'\ 22''$

2(ii): $\mathbf{58.7891°}$

58

$0.7891 \times 60 = 47.346$

$0.346 \times 60 = 20.76$

$58.7891° \approx 58°\ 47'\ 21''$

2(iii): $\mathbf{90.5678°}$

90

$0.5678 \times 60 = 34.068$

$0.068 \times 60 = 4.08$

$90.5678° \approx 90°\ 34' 4''$

3. Convert the following into decimal degrees:

(i) 65° 32' 15" (ii) 42° 18' 45" (iii) 78° 45' 36"

Solution

3(i): $\mathbf{65°32'15''}$

$65°32'15'' = 65 + \frac{32}{60} + \frac{15}{60\times60} = 65 + 0.5333 + 0.0042 = 65.5375°$

3(ii): $\mathbf{42°18'45''}$

$42°18'45'' = 42 + \frac{18}{60} + \frac{45}{60\times60} = 42 + 0.3 + 0.0125 = 42.3125°$

3(iii): $\mathbf{78°45'36''}$

$78°45'36'' = 78 + \frac{45}{60} + \frac{36}{60\times60} = 78 + 0.75 + 0.01 = 78.76°$

4. Convert the following into radians:

(i) 36° (ii) 22.5° (iii) 67.5°

Solution

4(i): $\mathbf{36°} = 36 \times \frac{\pi}{180} = \frac{\pi}{5}\text{rad}$

4(ii):$\mathbf{22.5°} = 22.5 \times \frac{\pi}{180} = \frac{\pi}{8}\text{rad}$

4(iii):$\mathbf{67.5°} = 67.5 \times \frac{\pi}{180} = \frac{3\pi}{8}\text{rad}$

5. Convert the following into degrees:

(i) $\frac{\pi}{16}$ rad (ii) $\frac{11\pi}{5}$ rad (iii) $\frac{7\pi}{6}$ rad

Solution

5(i): $\mathbf{\frac{\pi}{16}\text{rad}} = \frac{\pi}{16} \times \frac{180°}{\pi} = 11.25°$

5(ii): $\mathbf{\frac{11\pi}{5}\text{rad}} = \frac{11\pi}{5} \times \frac{180°}{\pi} = 396°$

5(iii): $\mathbf{\frac{7\pi}{6}\text{rad}} = \frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

6. Find the arc length and area of a sector if:

(i) $r = 6$ cm and central angle $\theta = \frac{\pi}{3}$ radians.

(ii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.

Solution

6(i): $\boldsymbol{l} = \text{r}\theta = 6 \times \frac{\pi}{3} = 6.28\text{cm}$

$\boldsymbol{A} = \frac{1}{2}\text{r}^2\theta = \frac{1}{2} \times (6)^2 \times \frac{\pi}{3} = 18.84\text{cm}^2$

6(ii): $\boldsymbol{l} = \text{r}\theta = \frac{4.8}{\pi} \times \frac{5\pi}{6} = 4\text{cm}$

$\boldsymbol{A} = \frac{1}{2}\text{r}^2\theta = \frac{1}{2} \times \left(\frac{4.8}{\pi}\right)^2 \times \frac{5\pi}{6} = 3.06\text{cm}^2$

7. If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.

Solution

$\theta = 60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}\text{rad}$

Area of the sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12)^2 \times \frac{\pi}{3} = 62.83\text{cm}^2$

Total area of the circle $= \pi r^2 = 3.14159 \times (12)^2 = 452.389\text{cm}^2$

Percentage $= \frac{\text{Area of the sector}}{\text{Total area of the circle}} \times 100\%$

Percentage $= \frac{62.83\text{cm}^2}{452.389\text{cm}^2} \times 100\% = 13.89\%$

8. Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

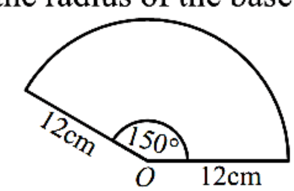
Solution

Percentage $= \frac{\text{Area of the sector}}{\text{Total area of the circle}} \times 100\%$

Percentage $= \frac{\theta}{2\pi} \times 100\% = \frac{\frac{\pi}{8}}{2\pi} \times 100\% = 6.25\%$

9. A circular sector of radius $r = 12$ cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?

Hint: Arc length of sector = circumference of cone.



Solution

Radius of the sector $= r = 12\text{cm}$

Angle of the sector $= \theta = 150° = 150 \times \frac{\pi}{180} = \frac{5\pi}{6}\text{rad}$

Arc Length $= l = r\theta = 12 \times \frac{5\pi}{6} = 10\pi\text{cm}$

Now

Circumference of base of the cone $= 2\pi r'$

$10\pi = 2\pi r'$

radius of base $= r' = 5\text{cm}$

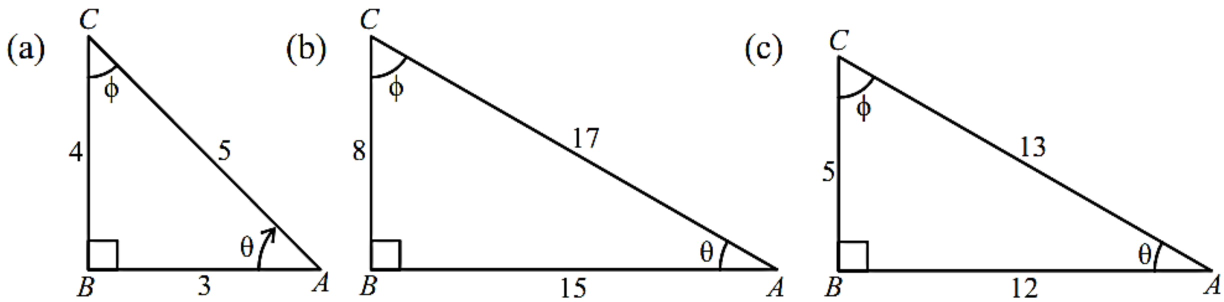
slant height $= l = r = 12\text{cm}$

EXERCISE 6.2

1. For each of the following right-angled triangles, find the trigonometric ratios:

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$ (iv) $\sec\theta$ (v) $\operatorname{cosec}\theta$

(vi) $\cot\phi$ (vii) $\tan\phi$ (viii) $\operatorname{cosec}\phi$ (ix) $\sec\phi$ (x) $\cos\phi$



Solution

(a) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{3}$ (iv) $\frac{5}{3}$ (v) $\frac{5}{4}$ (vi) $\frac{4}{3}$ (vii) $\frac{3}{4}$ (viii) $\frac{5}{3}$ (ix) $\frac{5}{4}$ (x) $\frac{4}{5}$

(b) (i) $\frac{8}{17}$ (ii) $\frac{15}{17}$ (iii) $\frac{8}{15}$ (iv) $\frac{17}{15}$ (v) $\frac{17}{8}$ (vi) $\frac{8}{15}$ (vii) $\frac{15}{8}$ (viii) $\frac{17}{15}$ (ix) $\frac{17}{8}$ (x) $\frac{8}{17}$

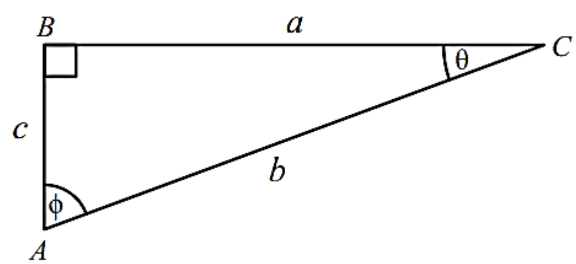
(c) (i) $\frac{5}{13}$ (ii) $\frac{12}{13}$ (iii) $\frac{5}{12}$ (iv) $\frac{13}{5}$ (v) $\frac{13}{12}$ (vi) $\frac{5}{12}$ (vii) $\frac{12}{5}$ (viii) $\frac{13}{12}$ (ix) $\frac{13}{5}$ (x) $\frac{5}{13}$

2. For the following right-angled triangle ABC find the trigonometric ratios for which $m\angle A = \phi$ and $m\angle C = \theta$

(i) $\sin\theta$ (ii) $\cos\theta$

(iii) $\tan\theta$ (iv) $\sin\phi$

(v) $\cos\phi$ (vi) $\tan\phi$



Solution

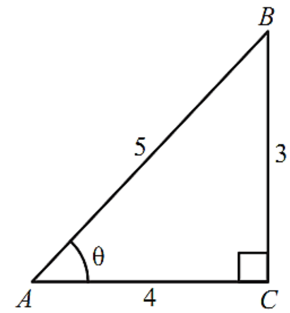
(i) $\frac{c}{b}$ (ii) $\frac{a}{b}$ (iii) $\frac{c}{a}$ (iv) $\frac{a}{b}$ (v) $\frac{c}{b}$ (vi) $\frac{a}{c}$

3. Considering the adjoining triangle ABC, verify that:

(i) $\sin\theta \ \text{cosec}\,\theta = 1$

(ii) $\cos\theta \ \sec\theta = 1$

(iii) $\tan\theta \ \cot\theta = 1$



Solution

3.(i) $sin\theta cosec\theta = \frac{3}{5} \times \frac{5}{3} = 1$

3.(ii) $cos\theta sec\theta = \frac{4}{5} \times \frac{5}{4} = 1$

3.(iii) $tan\theta cot\theta = \frac{3}{4} \times \frac{4}{3} = 1$

4. Fill in the blanks.

(i) $\sin 30° = \sin(90° - 60°) =$ $cos60°$

(ii) $\cos 30° = \cos(90° - 60°) =$ $sin60°$

(iii) $\tan 30° = \tan(90° - 60°) =$ $cot60°$

(iv) $\tan 60° = \tan(90° - 30°) =$ $cot30°$

(v) $\sin 60° = \sin(90° - 30°) =$ $cos30°$

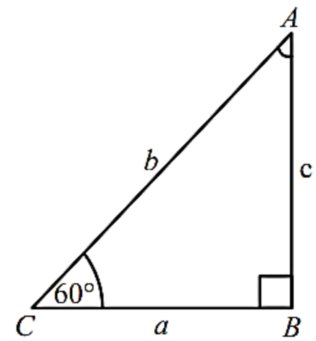
(vi) $\cos 60° = \cos(90° - 30°) =$ $sin30°$

(vii) $\sin 45° = \sin(90° - 45°) =$ $cos45°$

(viii) $\tan 45° = \tan(90° - 45°) =$ $cot45°$

(ix) $\cos 45° = \cos(90° - 45°) =$ $sin45°$

5. In a right angled triangle ABC, $m\angle B = 90°$ and C is an acute angle of 60°. Alsc $\sin m\angle A = \frac{a}{b}$, then find the following trigonometric ratios:



(i) $\frac{m\overline{BC}}{m\overline{AB}}$ (ii) $\cos 60°$

(iii) $\tan 60°$ (iv) $\text{cosec}\ \frac{\pi}{3}$

(v) $\cot 60°$ (vi) $\sin 30°$

(vii) $\cos 30°$ (viii) $\tan \frac{\pi}{6}$

(ix) $\sec 30°$ (x) $\cot 30°$

Solution

(i) $\frac{a}{c}$ (ii) $\frac{a}{b}$ (iii) $\frac{c}{a}$ (iv) $\frac{b}{c}$ (v) $\frac{a}{c}$

(vi) $\frac{a}{b}$ (vii) $\frac{c}{b}$ (viii) $\frac{a}{c}$ (ix) $\frac{b}{c}$ (x) $\frac{c}{a}$

EXERCISE 6.3

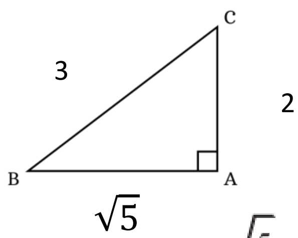
1. If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

(i) $\sin\theta = \frac{2}{3}$ (ii) $\cos\theta = \frac{3}{4}$ (iii) $\tan\theta = \frac{1}{2}$

(iv) $\sec\theta = 3$ (v) $\cot\theta = \sqrt{\frac{3}{2}}$

Solution

1.(i) $\boldsymbol{\sin\theta = \frac{2}{3}}$



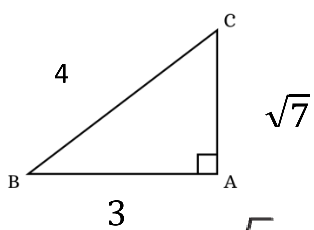
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 3^2 = 2^2 + B^2$$

$$\Rightarrow B^2 = 9 - 4 = 5 \Rightarrow B = \sqrt{5}$$

(i) $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{2}{\sqrt{5}}$, $\operatorname{cosec}\theta = \frac{3}{2}$, $\sec\theta = \frac{3}{\sqrt{5}}$, $\cot\theta = \frac{\sqrt{5}}{2}$

1.(ii) $\boldsymbol{\cos\theta = \frac{3}{4}}$



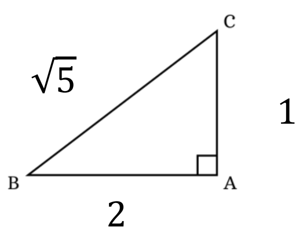
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 4^2 = P^2 + 3^2$$

$$\Rightarrow P^2 = 16 - 9 = 7 \Rightarrow P = \sqrt{7}$$

(ii) $\sin\theta = \frac{\sqrt{7}}{4}$, $\tan\theta = \frac{\sqrt{7}}{3}$, $\operatorname{cosec}\theta = \frac{4}{\sqrt{7}}$, $\sec\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{\sqrt{7}}$

1.(iii) $\boldsymbol{\tan\theta = \frac{1}{2}}$



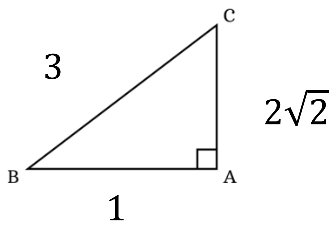
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow H^2 = 1^2 + 2^2$$

$$\Rightarrow H^2 = 1 + 4 = 5 \Rightarrow H = \sqrt{5}$$

(iii) $\sin\theta = \frac{1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$, $\operatorname{cosec}\theta = \sqrt{5}$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = 2$

1.(iv) $\boldsymbol{sec\theta = 3 = \frac{3}{1}}$



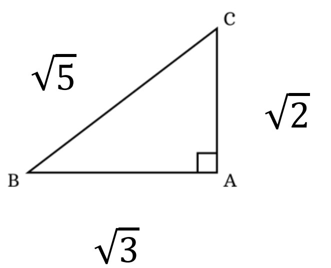
By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow 3^2 = P^2 + 1^2$$

$$\Rightarrow P^2 = 9 - 1 = 8 \Rightarrow P = 2\sqrt{2}$$

(iv) $\sin\theta = \dfrac{2\sqrt{2}}{3}$, $\cos\theta = \dfrac{1}{3}$, $\tan\theta = 2\sqrt{2}$, $\operatorname{cosec}\theta = \dfrac{3}{2\sqrt{2}}$, $\cot\theta = \dfrac{1}{2\sqrt{2}}$

1.(v) $\boldsymbol{cot\theta = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}}}$



By Pythagoras Formula

$$H^2 = P^2 + B^2 \Rightarrow H^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2$$

$$\Rightarrow H^2 = 2 + 3 = 5 \Rightarrow H = \sqrt{5}$$

(v) $\sin\theta = \sqrt{\dfrac{2}{5}}$, $\cos\theta = \sqrt{\dfrac{3}{5}}$, $\tan\theta = \sqrt{\dfrac{2}{3}}$, $\operatorname{cosec}\theta = \sqrt{\dfrac{5}{2}}$, $\sec\theta = \sqrt{\dfrac{5}{3}}$

Prove the Following Trigonometric Identities

2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Solution

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

3. $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

Solution

$\dfrac{\cos\theta}{\sin\theta} = \cot\theta = \dfrac{1}{\tan\theta}$

4. $$\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=1$$

Solution

$$\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=\sin\theta\times\frac{1}{\operatorname{cosec}\theta}+\cos\theta\times\frac{1}{\sec\theta}$$

$$\frac{\sin\theta}{\operatorname{cosec}\theta}+\frac{\cos\theta}{\sec\theta}=\sin\theta\times\sin\theta+\cos\theta\times\cos\theta=\sin^2\theta+\cos^2\theta=1$$

5. $$\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$$

Solution

$$\cos^2\theta-\sin^2\theta=\cos^2\theta-(1-\cos^2\theta)=\cos^2\theta-1+\cos^2\theta$$

$$\cos^2\theta-\sin^2\theta=2\cos^2\theta-1$$

6. $$\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$$

Solution

$$\cos^2\theta-\sin^2\theta=(1-\sin^2\theta)-\sin^2\theta=1-\sin^2\theta-\sin^2\theta$$

$$\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$$

7. $$\frac{1-\sin\theta}{\cos\theta}=\frac{\cos\theta}{1+\sin\theta}$$

Solution

$$\frac{1-\sin\theta}{\cos\theta}=\frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\cos\theta}{1+\sin\theta}$$

8. $$(\sec\theta-\tan\theta)^2=\frac{1-\sin\theta}{1+\sin\theta}$$

Solution

$$(\sec\theta-\tan\theta)^2=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2=\frac{(1-\sin\theta)^2}{\cos^2\theta}=\frac{(1-\sin\theta)(1-\sin\theta)}{1-\sin^2\theta}$$

$$(\sec\theta-\tan\theta)^2=\frac{(1-\sin\theta)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\frac{1-\sin\theta}{1+\sin\theta}$$

9. $$(\tan\theta+\cot\theta)^2=\sec^2\theta\operatorname{cosec}^2\theta$$

Solution

$$(\tan\theta+\cot\theta)^2=\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)^2=\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)^2=\left(\frac{1}{\cos\theta\sin\theta}\right)^2$$

$$(\tan\theta+\cot\theta)^2=\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}=\sec^2\theta\operatorname{cosec}^2\theta$$

$$10. \quad \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

Solution

$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{\tan\theta + \sec\theta - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]}{1 - \sec\theta + \tan\theta}$

$= \tan\theta + \sec\theta$

$$11. \quad \sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

Solution

$\sin^3\theta - \cos^3\theta$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$$12. \quad \sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$$

Solution

$\sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta]$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

EXERCISE 6.4

θ	0	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

1. Find the value of the following trigonometric ratios without using the calculator.

(i) sim 30° (ii) cos 30° (iii) $\tan\frac{\pi}{6}$ (iv) tan 60°

(v) sec 60° (vi) $\cos\frac{\pi}{3}$ (vii) cot 60° (viii) sin 60°

(ix) sec 30° (x) cosec 30° (xi) sin 45° (xii) $\cos\frac{\pi}{4}$

Solution

(i) $\frac{1}{2}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{\sqrt{3}}{3}$ (iv) $\sqrt{3}$

(v) 2 (vi) $\frac{1}{2}$ (vii) $\frac{\sqrt{3}}{3}$ (viii) $\frac{\sqrt{3}}{2}$

(ix) $\frac{2\sqrt{3}}{3}$ (x) 2 (xi) $\frac{\sqrt{2}}{2}$ (xii) $\frac{\sqrt{2}}{2}$

2. Evaluate:

(i) $2\sin 60^\circ \cos 60^\circ$ (ii) $2\cos\dfrac{\pi}{3}\sin\dfrac{\pi}{3}$

(iii) $2\sin 45^\circ + 2\cos 45^\circ$ (iv) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(v) $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$ (vi) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(vii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$ (viii) $\tan\dfrac{\pi}{6}\cot\dfrac{\pi}{6} + 1$

Solution

2(i): $2\sin60°\cos60° = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

2(ii): $2\cos\frac{\pi}{3}\sin\frac{\pi}{3} = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

2(iii): $2\sin45° + 2\cos45° = 2 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

2(iv): $\sin60°\cos30° + \cos60°\sin30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

2(v): $\cos60°\cos30° - \sin60°\sin30° = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

2(vi): $\sin60°\cos30° - \cos60°\sin30° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

2(vii): $\cos60°\cos30° + \sin60°\sin30° = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

2(viii): $\tan\frac{\pi}{6}\cot\frac{\pi}{6} + 1 = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{1} + 1 = 1 + 1 = 2$

3. If $\sin\dfrac{\pi}{4}$ and $\cos\dfrac{\pi}{4}$ equal to $\dfrac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) $2\sin 45^\circ - 2\cos 45^\circ$ (ii) $3\cos 45^\circ + 4\sin 45^\circ$

(iii) $5\cos 45^\circ - 3\sin 45^\circ$

Solution

3(i): $2\sin45° - 2\cos45° = 2 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} - \sqrt{2} = 0$

3(ii): $3\cos45° + 4\sin45° = 3 \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

3(iii): $5\cos45° - 3\sin45° = 5 \times \frac{1}{\sqrt{2}} - 3 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

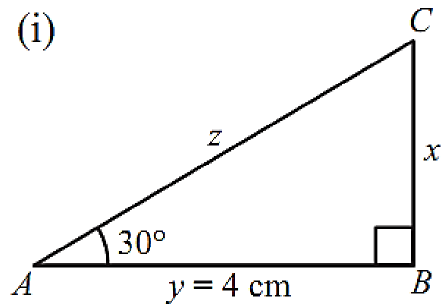
EXERCISE 6.5

1. Find the values of x, y and z from the following right angled triangles.

1(i) $m\angle A = 30°, y = 4cm$

Solution

(i)



$m\angle C = m\angle B - m\angle A = 90° - 30°$

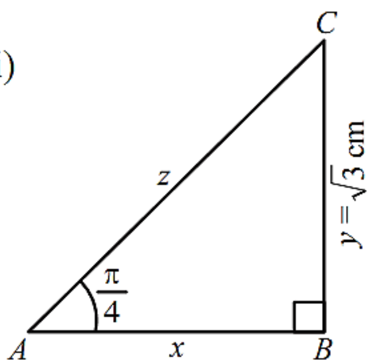
$m\angle C = 60°$

$\frac{x}{y} = \tan 30°$ $\frac{x}{4} = \frac{1}{\sqrt{3}}$ $x = \frac{4}{\sqrt{3}}$	$\frac{y}{z} = \cos 30°$ $\frac{4}{z} = \frac{\sqrt{3}}{2}$ $z = 4 \times \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

1(ii) $m\angle A = 45°, y = \sqrt{3}cm$

Solution

(ii)



$m\angle C = m\angle B - m\angle A = 90° - 45°$

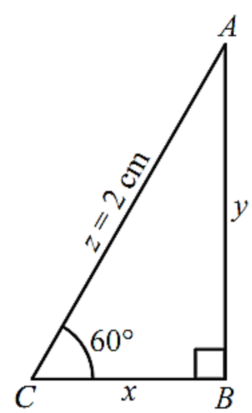
$m\angle C = 45°$

$\frac{y}{x} = \tan 45°$ $\frac{\sqrt{3}}{x} = 1$ $x = \sqrt{3}$	$\frac{y}{z} = \sin 45°$ $\frac{\sqrt{3}}{z} = \frac{1}{\sqrt{2}}$ $z = \sqrt{3} \times \sqrt{2} = \sqrt{6}$

1(iii) $m\angle C = 60°, z = 2cm$

Solution

(iii)



$m\angle A = m\angle B - m\angle C = 90° - 60°$

$m\angle A = 30°$

$\frac{x}{z} = \cos 60°$ $\frac{x}{2} = \frac{1}{2}$ $x = \frac{2}{2}$ $x = 1$	$\frac{y}{z} = \sin 60°$ $\frac{y}{2} = \frac{\sqrt{3}}{2}$ $y = \frac{2 \times \sqrt{3}}{2}$ $y = \sqrt{3}$

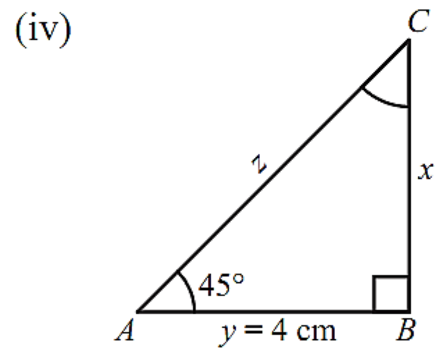
1(iv) $m\angle A = 45°, y = 4cm$

Solution

$m\angle C = m\angle B - m\angle A = 90° - 45°$

$m\angle C = 45°$

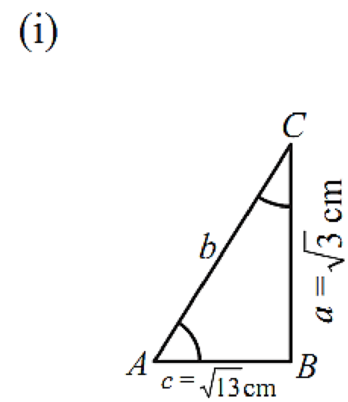
$\frac{x}{y} = \tan 45°$ $\frac{x}{4} = 1$ $x = 4$	$\frac{y}{z} = \cos 45°$ $\frac{4}{z} = \frac{1}{\sqrt{2}}$ $z = 4\sqrt{2}$



2. Find the unknown side and angles of the following triangles.

2(i)

By Pythagoras Formula



$b^2 = a^2 + c^2 \Rightarrow b^2 = (\sqrt{3})^2 + (\sqrt{13})^2$

$\Rightarrow b^2 = 3 + 13 = 16 \Rightarrow b = 4$

$\sin A = \frac{a}{b} = \frac{\sqrt{3}}{4} = 0.4330$

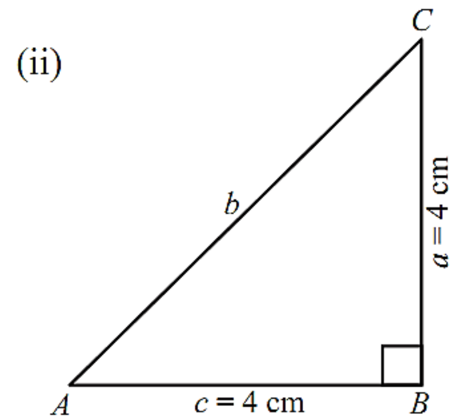
$A = \sin^{-1}(0.4330) = 25.64°$

$m\angle C = m\angle B - m\angle A = 90° - 25.64°$

$m\angle C = 64.36°$

2(ii)

By Pythagoras Formula



$b^2 = a^2 + c^2 \Rightarrow b^2 = (4)^2 + (4)^2$

$\Rightarrow b^2 = 16 + 16 = 32 \Rightarrow b = 4\sqrt{2}$

$\cos A = \frac{c}{b} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.7071$

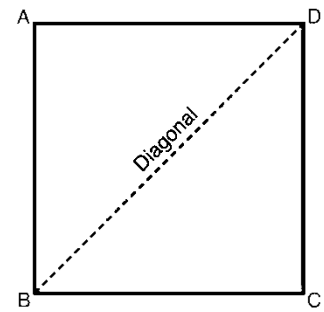
$A = \sin^{-1}(0.7071) = 45°$

$m\angle C = m\angle B - m\angle A = 90° - 45°$

$m\angle C = 45°$

3. Each side of a square field is 60 m long. Find the lengths of the diagonals of the field.

Solution



A square's diagonal forms a right-angled triangle with two sides.
If 'a' and 'b' are the sides of the square and 'c' is the diagonal. Then
Using Pythagorean Theorem:

$c^2 = a^2 + b^2$

In this case, a = b = 60m.

Therefore, $c^2 = 60^2 + 60^2$

$c^2 = 3600 + 3600 = 7200$

$c = \sqrt{7200} = \sqrt{3600 \times 2} = 60\sqrt{2}$m

Solve the following triangles when $m\angle B = 90°$:

4. $m\angle C = 60°,\ c = 3\sqrt{3}$ cm

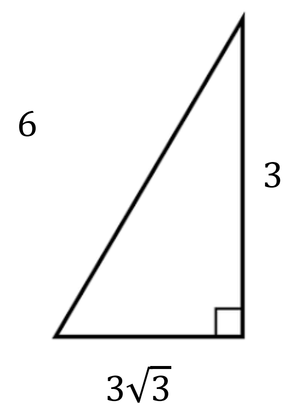
Solution

$m\angle C = 60°, c = 3\sqrt{3}\text{cm}$

$m\angle A = m\angle B - m\angle C = 90° - 60°$

$m\angle A = 30°$

$\frac{c}{b} = \sin 60°$ $\frac{3\sqrt{3}}{b} = \frac{\sqrt{3}}{2}$ $b = \frac{2\times 3\sqrt{3}}{\sqrt{3}}$ $b = 6\text{cm}$	$\frac{a}{b} = \sin 30°$ $\frac{a}{6} = \frac{1}{2}$ $a = \frac{6}{2}$ $a = 3\text{cm}$



Solve the following triangles when $m\angle B = 90°$:

5. $m\angle C = 45°,\ a = 8$ cm

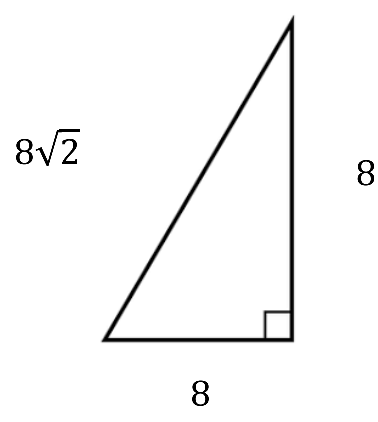
Solution

$m\angle C = 45°, a = 8\text{cm}$

$m\angle A = m\angle B - m\angle C = 90° - 45°$

$m\angle A = 45°$

$\frac{a}{b} = \sin 45°$ $\frac{8}{b} = \frac{1}{\sqrt{2}}$ $b = 8\sqrt{2}\text{cm}$	$\frac{c}{b} = \cos 45°$ $\frac{c}{8\sqrt{2}} = \frac{1}{\sqrt{2}}$ $c = \frac{8\sqrt{2}}{\sqrt{2}}$ $c = 8\text{cm}$



Solve the following triangles when $m\angle B = 90°$:

6. $a = 12$ cm, $c = 6$ cm

Solution

By Pythagoras Formula

$b^2 = a^2 + c^2 \Rightarrow b^2 = (12)^2 + (6)^2$

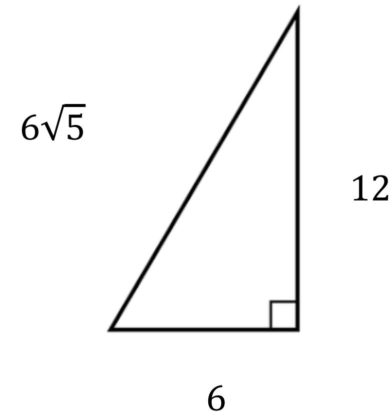
$\Rightarrow b^2 = 144 + 36 = 180 \Rightarrow b = 6\sqrt{5}$

$\sin A = \frac{a}{b} = \frac{12}{6\sqrt{5}} = 0.8944$

$A = \sin^{-1}(0.8944) = 63.4°$

$m\angle C = m\angle B - m\angle A = 90° - 63.4°$

$m\angle C = 26.6°$



Solve the following triangles when $m\angle B = 90°$:

7. $m\angle A = 60°$, $c = 4$ cm

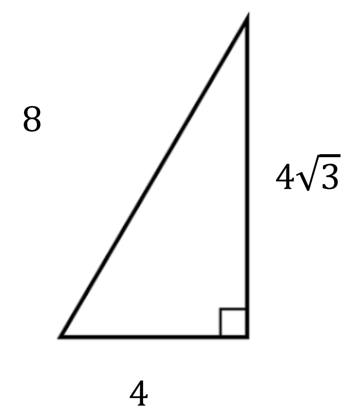
Solution

$m\angle A = 60°, c = 4\text{cm}$

$m\angle C = m\angle B - m\angle C = 90° - 60°$

$m\angle C = 30°$

$\frac{c}{b} = \cos 60°$ $\frac{4}{b} = \frac{1}{2}$ $b = 4 \times 2$ $b = 8\text{cm}$	$\frac{a}{b} = \sin 60°$ $\frac{a}{8} = \frac{\sqrt{3}}{2}$ $a = \frac{8\sqrt{3}}{2}$ $a = 4\sqrt{3}\text{cm}$



Solve the following triangles when $m\angle B = 90°$:

8. $m\angle A = 30°, c = 4cm$ **wrong statement in book**

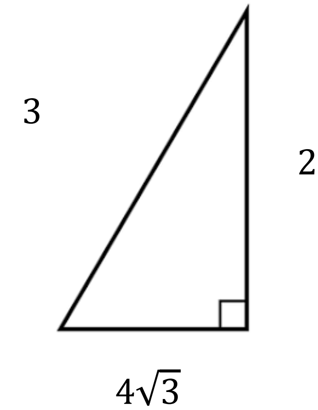
Solution

$m\angle A = 30°, c = 4cm$

$m\angle C = m\angle B - m\angle C = 90° - 30°$

$m\angle C = 60°$

$\frac{c}{b} = \cos 60°$ $\frac{4}{b} = \frac{1}{2}$ $b = 4 \times 2$ $b = 8cm$	$\frac{a}{b} = \sin 60°$ $\frac{a}{8} = \frac{\sqrt{3}}{2}$ $a = \frac{8\sqrt{3}}{2}$ $a = 4\sqrt{3}cm$



Solve the following triangles when $m\angle B = 90°$:

9. $b = 10$ cm, $a = 6$ cm

Solution

By Pythagoras Formula

$b^2 = a^2 + c^2 \Rightarrow (10)^2 = c^2 + (6)^2$

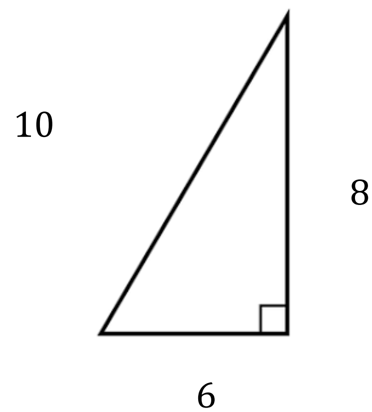
$\Rightarrow c^2 = 100 - 36 = 64 \Rightarrow c = 8$

$\sin C = \frac{c}{b} = \frac{8}{10} = 0.8$

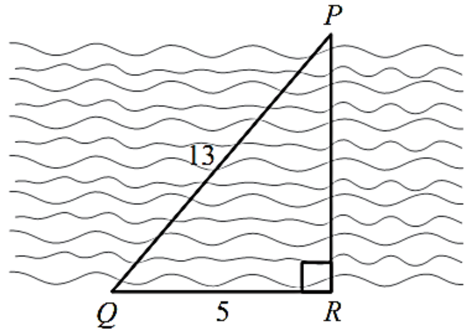
$C = \sin^{-1}(0.8) = 53.1°$

$m\angle A = m\angle B - m\angle C = 90° - 53.1°$

$m\angle A = 36.9°$



10. Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR.



Solution

By Pythagoras Formula

$|PQ|^2 = |QR|^2 + |PR|^2$

$(13)^2 = (5)^2 + |PR|^2$

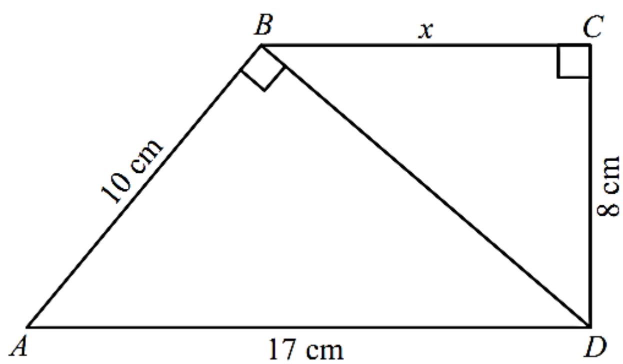
$|PR|^2 = 169 - 25 = 144$

$|PR| = 12km$

$\tan(\angle \mathrm{PQR}) = \frac{\mathrm{PR}}{\mathrm{QR}} = \frac{12}{5} = 2.4$

$\angle \mathrm{PQR} = \tan^{-1}(2.4) = 67.38°$

11. Calculate the length x in the adjoining figure.



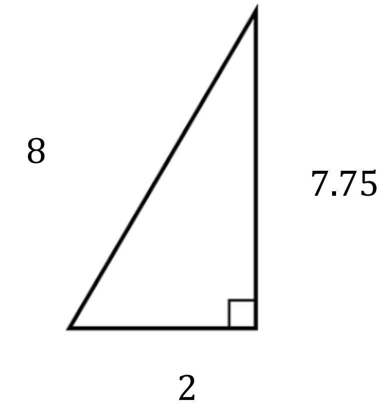
Solution

Applying Pythagoras Formula For ΔABD $\lvert AD\rvert^2 = \lvert BD\rvert^2 + \lvert AB\rvert^2$ $(17)^2 = \lvert BD\rvert^2 + (10)^2$ $\lvert BD\rvert^2 = 289 - 100 = 189$ $\lvert BD\rvert = 3\sqrt{21}$	Again applying Pythagoras Formula For ΔBCD $\lvert BD\rvert^2 = \lvert BC\rvert^2 + \lvert CD\rvert^2$ $\left(3\sqrt{21}\right)^2 = x^2 + (8)^2$ $x^2 = 189 - 64 = 125$ $x = 5\sqrt{5}$

12. If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall.

Solution

By Pythagoras Formula



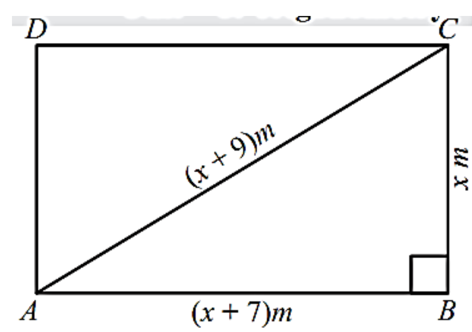
$8^2 = H^2 + 2^2$

$64 = H^2 + 4$

$H^2 = 64 - 4 = 60$

$H = 7.75m$

13. The diagonal of a rectangular field $ABCD$ is $(x + 9)m$ and the sides are $(x + 7)$m and x m. Find the value of x.



Solution

By Pythagoras Formula

$(x + 9)^2 = (x + 7)^2 + x^2$

$x^2 + 18x + 81 = x^2 + 14x + 49 + x^2$

$x^2 + 18x + 81 = 2x^2 + 14x + 49$

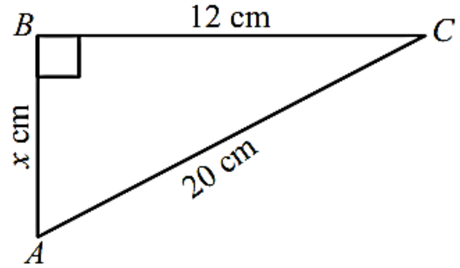
$x^2 - 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$x = 8$ or $x = -4$

Since x cannot be negative, therefore $x = 8$

14. Calculate the value of ‘x’ in each case.



Solution

By Pythagoras Formula

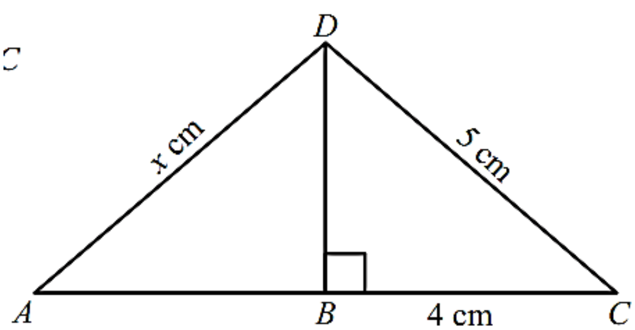
$|AC|^2 = |BC|^2 + |AB|^2$

$(20)^2 = (12)^2 + x^2$

$x^2 = 400 - 144 = 256$

$x = 16cm$

14. Calculate the value of ‘x’ in each case.



$m\overline{AB} = m\overline{BC}$

Solution

Applying Pythagoras Formula For ΔDBC $\lvert DC\rvert^2 = \lvert DB\rvert^2 + \lvert BC\rvert^2$ $(5)^2 = \lvert DB\rvert^2 + (4)^2$ $\lvert DB\rvert^2 = 25 - 16 = 9$ $\lvert DB\rvert = 3cm$	Again applying Pythagoras Formula For ΔDBA $\lvert AD\rvert^2 = \lvert DB\rvert^2 + \lvert AB\rvert^2$ $x^2 = (3)^2 + (4)^2$ $x^2 = 9 + 16 = 25$ $x = 5\ cm$