Al Huda Science Academy Notes

10th Mathematics — New Book
Chapter 1: Complex Numbers

Exercise 1.1 Full Solution

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Board Exam Preparation Series

Q.1 — Simplify the following:

(i) i^5

$i^5 = i^4 \times i = 1 \times i$

$i^5 = i$

(ii) i^{16}

$i^{16} = (i^4)^4 = 1^4$

$i^{16} = 1$

(iii) $(-i)^{-19}$

$(-i)^{-19} = (1/-i)^{19} = (-i)^{19}$

$= (-i)^{16} \times (-i)^3 = (1)^4 \times (-i)^3 = -(-i^2 \times i) = -i$

$(-i)^{-19} = -i$

(iv) $27i^{-26}$

$27i^{-26} = 27 \times 1/i^{26} = 27 \times 1/[(i^4)^6 \times i^2]$

$= 27 \times 1/(1^6 \times i^2) = 27 \times 1/i^2 = 27 \times (-1)$

$27i^{-26} = -27$

(v) $i^{11} + i^5$

$i^{11} + i^5 = i^{8+3} + i^4 \cdot i = (i^4)^2 \cdot i^3 + i$

$= 1 \cdot (-i) + i = -i + i$

$i^{11} + i^5 = 0$

(vi) $(i^4 + i^3 + i^2 + i)^2$

Inside the bracket:

$i^4 + i^3 + i^2 + i = 1 + (-i) + (-1) + i = (1-1) + (-i+i) = 0$

$(0)^2$

$(i^4 + i^3 + i^2 + i)^2 = 0$

(vii) $(i^8/i^7)^{-5}$

$(i^8/i^7)^{-5} = (i^{8-7})^{-5} = (i^1)^{-5} = (1/i^3)^5 = (1/-i)^5$

$= (-1)^{-5} \times 1/i^5 = 1/i^4 \cdot i = 1/i$

$(i^8/i^7)^{-5} = i$

(viii) $i^{13} \times i^{29}$

$i^{13} \times i^{29} = i^{13+29} = i^{42} = i^{4 \cdot 10+2} = (i^4)^{10} \times i^2 = 1^{10} \times (-1)$

$i^{13} \times i^{29} = -1$

Q.2 — Write in terms of i:

(i) $2 + \sqrt{-4}$

$2 + \sqrt{-4} = 2 + \sqrt{(-1)(4)} = 2 + \sqrt{-1} \cdot \sqrt{4} = 2 + i \cdot 2$

$$\boxed{2 + \sqrt{-4} = 2 + 2i}$$

(ii) $3 - \sqrt{-7}$

$3 - \sqrt{-7} = 3 - \sqrt{(-1)(7)} = 3 - \sqrt{-1} \cdot \sqrt{7} = 3 - i\sqrt{7}$

$$\boxed{3 - \sqrt{-7} = 3 - i\sqrt{7}}$$

(iii) $2/5 + \sqrt{-16}/\sqrt{5}$

$2/5 + \sqrt{-16}/\sqrt{5} = 2/5 + [\sqrt{(-1)(16)}]/\sqrt{5} = 2/5 + (i\sqrt{16})/\sqrt{5} = 2/5 + 4i/5$

$$\boxed{= (2 + 4i)/5}$$

(iv) $\sqrt{2} - \sqrt{-3}$

$\sqrt{2} - \sqrt{-3} = \sqrt{2} - \sqrt{(-1)(3)} = \sqrt{2} - \sqrt{-1} \cdot \sqrt{3}$

$$\boxed{\sqrt{2} - \sqrt{-3} = \sqrt{2} - i\sqrt{3}}$$

Q.3 — Find the values of x and y:

(i) $(2x + 5) + (y - 3)i = 1 + 2i$

Equate real parts:

$2x + 5 = 1 \Rightarrow 2x = -4 \Rightarrow x = -2$

Equate imaginary parts:

$y - 3 = 2 \Rightarrow y = 5$

$$\boxed{x = -2,\ y = 5}$$

(ii) $(3x + 2) - (4 - y)i = 5 + 3i$

Equate real parts:

$3x + 2 = 5 \Rightarrow 3x = 3 \Rightarrow x = 1$

Equate imaginary parts:

$-(4 - y) = 3 \Rightarrow -4 + y = 3 \Rightarrow y = 7$

$$\boxed{x = 1,\ y = 7}$$

(iii) $(2 + i)x + (1 - 2i)y = 3 + 4i$

Expanding:

$2x + ix + y - 2iy = 3 + 4i \rightarrow (2x + y) + (x - 2y)i = 3 + 4i$

Equate real and imaginary parts:

$2x + y = 3 \quad ...(1)$

$x - 2y = 4 \quad ...(2)$

From (1): y = 3 − 2x. Substitute into (2):

$x - 2(3 - 2x) = 4 \Rightarrow x - 6 + 4x = 4 \Rightarrow 5x = 10 \Rightarrow x = 2$

$y = 3 - 2(2) = 3 - 4 = -1$

$x = 2,\ y = -1$

(iv) $(1 - i)x + (2 + i)y = 4 - i$

Expanding:

$x - ix + 2y + iy = 4 - i \rightarrow (x + 2y) + (-x + y)i = 4 - i$

Equate real and imaginary parts:

$x + 2y = 4 \quad ...(1)$

$-x + y = -1 \quad ...(2)$

From (1): x = 4 − 2y. Substitute into (2):

$-(4 - 2y) + y = -1 \Rightarrow -4 + 2y + y = -1 \Rightarrow 3y = 3 \Rightarrow y = 1$

$x = 4 - 2(1) = 2$

$x = 2,\ y = 1$

(v) $(3x - 1) + (2y - 3)i = 8 + 7i$

Equate real parts:

$3x - 1 = 8 \Rightarrow 3x = 9 \Rightarrow x = 3$

Equate imaginary parts:

$2y - 3 = 7 \Rightarrow 2y = 10 \Rightarrow y = 5$

$x = 3,\ y = 5$

KEY FACTS — Exercise 1.1

Powers of i (cycle repeats every 4)

$i^0 = 1$ $i^1 = i$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$

$i^5 = i$ $i^6 = -1$ $i^7 = -i$ $i^8 = 1$

Important index rules

$i^m \cdot i^n = i^{m+n}$

$i^m / i^n = i^{m-n}$

$(i^m)^n = i^{mn}$

$i^{-n} = 1/i^n$

Square roots of negatives

$\sqrt{(-a)} = i\sqrt{a} = (\sqrt{a})\cdot i$ (where $a > 0$)

Conjugate rules

If $z = a + bi$, then $\bar{z} = a - bi$

$z + \bar{z} = 2a$ $z - \bar{z} = 2bi$