Al Huda Science Academy Notes

10th Mathematics — New Book
Chapter 1: Complex Numbers

Exercise 1.2 Full Solution

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Board Exam Preparation Series

Q.1 — Simplify and write in the form a + bi:

(i) (2 + 5i) + (3 − 2i)

$= 2 + 5i + 3 - 2i = (2 + 3) + (5i - 2i)$

$= 5 + 3i$

(ii) (16 − 3i) + (9 + 2i)

$= 16 - 3i + 9 + 2i = (16 + 9) + (-3i + 2i)$

$= 25 - i$

(iii) (9 − 2i) − (7 − 3i)

$= 9 - 2i - 7 + 3i = (9 - 7) + (-2i + 3i)$

$= 2 + i$

(iv) (11 + 9i) − (9 − 7i)

$= 11 + 9i - 9 + 7i = (11 - 9) + (9i + 7i)$

$= 2 + 16i$

(v) (3 + 4i)(2 − 3i)

$= 3(2 - 3i) + 4i(2 - 3i) = 6 - 9i + 8i - 12i^2$

$= 6 - 9i + 8i - 12(-1) = 6 - i + 12$

$= 18 - i$

(vi) (5 − 2i)(3 − 4i)

$= 5(3 - 4i) - 2i(3 - 4i) = 15 - 20i - 6i + 8i^2$

$= 15 - 26i + 8(-1) = 15 - 26i - 8$

$= 7 - 26i$

(vii) (3 − 5i) ÷ (2 − 4i)

Multiply numerator and denominator by the conjugate (2 + 4i):

$= [(3 - 5i)(2 + 4i)] / [(2 - 4i)(2 + 4i)] = (6 + 12i - 10i - 20i^2) / (4 - 16i^2)$

$= (6 + 2i + 20) / (4 + 16) = (26 + 2i) / 20$

$= 13/10 + (1/10)i$

(viii) (5 + 2i) ÷ (6 − 3i)

Multiply numerator and denominator by the conjugate (6 + 3i):

$= [(5 + 2i)(6 + 3i)] / [(6 - 3i)(6 + 3i)] = (30 + 15i + 12i + 6i^2) / (36 - 9i^2)$

$= (30 + 27i + 6(-1)) / (36 + 9) = (24 + 27i) / 45$

$= 8/15 + (3/5)i$

Q.2 — Write the additive inverse for each complex number:

(i) $3 + 2i$

Additive inverse = $-(3 + 2i)$

$= -3 - 2i$

(ii) $4 - 3i$

Additive inverse = $-(4 - 3i)$

$= -4 + 3i$

(iii) $5 - 7i$

Additive inverse = $-(5 - 7i)$

$= -5 + 7i$

(iv) $-2/3 + (5/4)i$

Additive inverse = $-(-2/3 + (5/4)i)$

$= 2/3 - (5/4)i$

Q.3 — Find the multiplicative inverse for each complex number:

(i) $4 + 5i$

Inverse = $1/(4 + 5i) \times (4 - 5i)/(4 - 5i) = (4 - 5i)/(16 + 25)$

$= (4 - 5i)/41$

(ii) $6 + 2i$

Inverse = $1/(6 + 2i) \times (6 - 2i)/(6 - 2i) = (6 - 2i)/(36 + 4) = (6 - 2i)/40 = 2(3 - i)/40$

$= (3 - i)/20$

(iii) $7 - 3i$

Inverse = $1/(7 - 3i) \times (7 + 3i)/(7 + 3i) = (7 + 3i)/(49 + 9)$

$= (7 + 3i)/58$

(iv) $\sqrt{5} - 4i$

Inverse = $1/(\sqrt{5} - 4i) \times (\sqrt{5} + 4i)/(\sqrt{5} + 4i) = (\sqrt{5} + 4i)/((\sqrt{5})^2 + 4^2) = (\sqrt{5} + 4i)/(5 + 16)$

$= (\sqrt{5} + 4i)/21$

Q.4 — If $z_1 = 2 + 5i$, $z_2 = 1 - 3i$ and $z_3 = 2 + i$, then verify that:

(i) $z_1 + z_2 = z_2 + z_1$

$L.H.S = z_1 + z_2 = (2 + 5i) + (1 - 3i) = 2 + 5i + 1 - 3i = 3 + 2i$

$R.H.S = z_2 + z_1 = (1 - 3i) + (2 + 5i) = 1 - 3i + 2 + 5i = 3 + 2i$

L.H.S = R.H.S = 3 + 2i ✓ Verified

(ii) $z_1z_2 = z_2z_1$

$L.H.S = z_1z_2 = (2 + 5i)(1 - 3i) = 2(1 - 3i) + 5i(1 - 3i) = 2 - 6i + 5i - 15i^2$

$= 2 - i - 15(-1) = 17 - i$

$R.H.S = z_2z_1 = (1 - 3i)(2 + 5i) = 1(2 + 5i) - 3i(2 + 5i) = 2 + 5i - 6i - 15i^2$

$= 2 - i + 15 = 17 - i$

L.H.S = R.H.S = 17 − i ✓ Verified

(iii) $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

$L.H.S = (z_1 + z_2) + z_3 = (2 + 5i + 1 - 3i) + (2 + i) = (3 + 2i) + (2 + i) = 5 + 3i$

$R.H.S = z_1 + (z_2 + z_3) = (2 + 5i) + ((1 - 3i) + (2 + i)) = (2 + 5i) + (3 - 2i) = 5 + 3i$

L.H.S = R.H.S = 5 + 3i ✓ Verified

(iv) $(z_1z_2)z_3 = z_1(z_2z_3)$

L.H.S = $(z_1z_2)z_3$ — First $z_1z_2 = 17 - i$ (from part ii). Then:

$(17 - i)(2 + i) = 17(2 + i) - i(2 + i) = 34 + 17i - 2i - i^2$

$= 34 + 15i + 1 = 35 + 15i$

R.H.S = $z_1(z_2z_3)$ — First z_2z_3:

$z_2z_3 = (1 - 3i)(2 + i) = 1(2 + i) - 3i(2 + i) = 2 + i - 6i - 3i^2 = 2 - 5i + 3 = 5 - 5i$

$z_1(z_2z_3) = (2 + 5i)(5 - 5i) = 2(5 - 5i) + 5i(5 - 5i) = 10 - 10i + 25i - 25i^2$

$= 10 + 15i + 25 = 35 + 15i$

L.H.S = R.H.S = 35 + 15i ✓ Verified

(v) $z_1 + (-z_1) = (-z_1) + z_1 = 0$

$L.H.S = z_1 + (-z_1) = (2 + 5i) + (-2 - 5i) = 2 + 5i - 2 - 5i = 0$

$R.H.S = (-z_1) + z_1 = (-2 - 5i) + (2 + 5i) = -2 - 5i + 2 + 5i = 0$

L.H.S = R.H.S = 0 ✓ Verified

Q.5 — If $(1 + i)^2/(2 - i) = x + iy$, then find the values of x and y.

Step 1: Simplify the numerator.

$(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$

Step 2: Divide and rationalize.

$2i/(2 - i) = 2i/(2 - i) \times (2 + i)/(2 + i) = 2i(2 + i)/(2^2 + 1^2)$

$= (4i + 2i^2)/5 = (4i - 2)/5 = -2/5 + (4/5)i$

$x = -2/5,\ y = 4/5$

Q.6 — If (2x + iy)(1 − i) = 4 + 2i, then find the values of x and y.

Step 1: Expand the left side.

$(2x + iy)(1 - i) = 2x(1 - i) + iy(1 - i) = (2x - 2xi) + (iy - i^2y)$

$= (2x + y) + (y - 2x)i$

Step 2: Equate to 4 + 2i, then compare real and imaginary parts.

Equate real parts: $2x + y = 4$ ...(1)

Equate imaginary parts: $y - 2x = 2$ ...(2)

Step 3: Add (1) and (2):

$2y = 6 \Rightarrow y = 3$

Substitute $y = 3$ in (1): $2x + 3 = 4 \Rightarrow 2x = 1 \Rightarrow x = 1/2$

$x = 1/2,\ y = 3$

Q.7 — Find the values of a and b, if (a + bi)(1 + 3i) = −8 + 11i.

Step 1: Expand the left side.

$(a + bi)(1 + 3i) = a(1 + 3i) + bi(1 + 3i) = a + 3ai + bi + 3bi^2$

$= (a - 3b) + (3a + b)i$

Step 2: Equate real and imaginary parts.

$a - 3b = -8$ ...(1)

$3a + b = 11$ ...(2)

Step 3: From (1), a = 3b − 8. Substitute in (2):

$3(3b - 8) + b = 11 \Rightarrow 9b - 24 + b = 11 \Rightarrow 10b = 35 \Rightarrow b = 7/2$

Step 4: Substitute b = 7/2 into a = 3b − 8:

$a = 3(7/2) - 8 = 21/2 - 16/2 = 5/2$

$a = 5/2,\ b = 7/2$

KEY FACTS — Exercise 1.2

Powers of i

$i^0=1$ $i^1=i$ $i^2=-1$ $i^3=-i$ $i^4=1$ (cycle repeats)

Conjugate & Modulus

If $z = a + bi$, then $\bar{z} = a - bi$

$|z| = \sqrt{(a^2 + b^2)}$

Multiplicative inverse

If $z = a + bi$ ($z \neq 0$), then $z^{-1} = 1/z = (a - bi)/(a^2 + b^2)$

Division of complex numbers

$(a + bi)/(c + di) \times (c - di)/(c - di) = [(a+bi)(c-di)] / (c^2 + d^2)$

Additive inverse

If $z = a + bi$, then $-z = -(a + bi) = -a - bi$