Al Huda Science Academy Notes

10th Mathematics — New Book
Chapter 1: Complex Numbers

Exercise 1.3 Full Solution

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Board Exam Preparation Series

Q.1 — Find the modulus of the following complex numbers:

(i) $4 + 3i$

$|z| = \sqrt{(4^2 + 3^2)} = \sqrt{(16 + 9)} = \sqrt{25}$

$|z| = 5$

(ii) $-5 - 4i$

$|z| = \sqrt{((-5)^2 + (-4)^2)} = \sqrt{(25 + 16)}$

$|z| = \sqrt{41}$

(iii) $3/5 - (4/5)i$

$|z| = \sqrt{((3/5)^2 + (-4/5)^2)} = \sqrt{(9/25 + 16/25)} = \sqrt{(25/25)}$

$|z| = 1$

(iv) $-\sqrt{2} - \sqrt{3}\, i$

$|z| = \sqrt{((-\sqrt{2})^2 + (-\sqrt{3})^2)} = \sqrt{(2 + 3)}$

$|z| = \sqrt{5}$

Q.2 — If $z_1 = 2 + 7i$ and $z_2 = 4 - 3i$, then verify that:

(i) $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

L.H.S: $z_1 + z_2 = (2 + 7i) + (4 - 3i) = 6 + 4i \rightarrow$ conjugate $= 6 - 4i$

R.H.S: $\bar{z}_1 + \bar{z}_2 = (2 - 7i) + (4 + 3i) = 6 - 4i$

L.H.S = R.H.S = $6 - 4i$ ✓ Verified

(ii) $\overline{(z_1 z_2)} = \bar{z}_1 \cdot \bar{z}_2$

L.H.S: $z_1 z_2 = (2 + 7i)(4 - 3i) = 8 - 6i + 28i - 21i^2 = 8 + 22i + 21 = 29 + 22i \rightarrow$ conjugate $= 29 - 22i$

R.H.S: $\bar{z}_1 \cdot \bar{z}_2 = (2 - 7i)(4 + 3i) = 8 + 6i - 28i - 21i^2 = 8 - 22i + 21 = 29 - 22i$

L.H.S = R.H.S = $29 - 22i$ ✓ Verified

(iii) $\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$

L.H.S:

$z_1/z_2 = (2 + 7i)/(4 - 3i) \times (4 + 3i)/(4 + 3i) = [(2+7i)(4+3i)] / (16 + 9)$

$= (8 + 6i + 28i + 21i^2)/25 = (-13 + 34i)/25 \rightarrow$ conjugate $= (-13 - 34i)/25$

R.H.S:

$\bar{z}_1/\bar{z}_2 = (2 - 7i)/(4 + 3i) \times (4 - 3i)/(4 - 3i) = [(2-7i)(4-3i)] / (16 + 9)$

$= (8 - 6i - 28i + 21i^2)/25 = (-13 - 34i)/25$

L.H.S = R.H.S = $(-13 - 34i)/25$ ✓ Verified

Q.3 — If $z = 5 - 2i$, then verify that:

(i) $\overline{(\bar{z})} = z$

$\bar{z} = 5 + 2i \rightarrow \overline{(\bar{z})} = \overline{(5 + 2i)} = 5 - 2i$

$\overline{(\bar{z})} = z = 5 - 2i$ ✓ Verified

(ii) $|z| = |\bar{z}|$

$|z| = \sqrt{(5^2 + (-2)^2)} = \sqrt{(25+4)} = \sqrt{29}$

$|\bar{z}| = \sqrt{(5^2 + 2^2)} = \sqrt{(25+4)} = \sqrt{29}$

$|z| = |\bar{z}| = \sqrt{29}$ ✓ Verified

(iii) $|z| = |-z|$

$-z = -(5 - 2i) = -5 + 2i \rightarrow |-z| = \sqrt{((-5)^2 + 2^2)} = \sqrt{(25+4)} = \sqrt{29}$

$|z| = |-z| = \sqrt{29}$ ✓ Verified

(iv) $\bar{z}z = |z|^2$

$\bar{z}z = (5 - 2i)(5 + 2i) = 25 - (2i)^2 = 25 + 4 = 29$

$|z|^2 = (\sqrt{29})^2 = 29$

$\bar{z}z = |z|^2 = 29$ ✓ Verified

(v) $|z| = |-\bar{z}|$

$-\bar{z} = -(5 + 2i) = -5 - 2i \rightarrow |-\bar{z}| = \sqrt{((-5)^2 + (-2)^2)} = \sqrt{(25+4)} = \sqrt{29}$

$|z| = |-\bar{z}| = \sqrt{29}$ ✓ Verified

Q.4 — If $z = 4 - 3i$, then verify that $|z| = |-z| = |\bar{z}| = |-\bar{z}|$.

$|z| = \sqrt{(4^2 + (-3)^2)} = \sqrt{(16 + 9)} = \sqrt{25} = 5$

$-z = -(4 - 3i) = -4 + 3i \Rightarrow |-z| = \sqrt{((-4)^2 + 3^2)} = \sqrt{25} = 5$

$\bar{z} = 4 + 3i \Rightarrow |\bar{z}| = \sqrt{(4^2 + 3^2)} = \sqrt{25} = 5$

$-\bar{z} = -4 - 3i \Rightarrow |-\bar{z}| = \sqrt{((-4)^2 + (-3)^2)} = \sqrt{25} = 5$

Therefore, $|z| = |-z| = |\bar{z}| = |-\bar{z}| = 5$ ✓ Verified

Q.5 — If $z_1 = 2 + 3i$, $z_2 = -1 + i$, then evaluate:

(i) $Re(z_1 z_2)$

$z_1z_2 = (2 + 3i)(-1 + i) = -2 + 2i - 3i + 3i^2 = -2 - i - 3 = -5 - i$

$Re(z_1z_2) = -5$

(ii) $Im(z_1z_2)$

From above, $z_1z_2 = -5 - i$

$Im(z_1z_2) = -1$

KEY FACTS — Exercise 1.3

Modulus

If $z = x + iy$, $|z| = \sqrt{(x^2 + y^2)}$

Conjugate

If $z = x + iy$, $\bar{z} = x - iy$

Basic identities

$\overline{(\bar{z})} = z$

$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

$\overline{(z_1z_2)} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$

$|z| = |\bar{z}| = |-z| = |-\bar{z}|$

$\bar{z}z = |z|^2$

Powers of i

$i^2=-1$ $i^3=-i$ $i^4=1$ $i^5=i$ $i^6=-1$

Algebraic rules

$(a+b)+c = a+(b+c)$

$a+b = b+a$

$a(bc)=(ab)c$

$a(b+c)=ab+ac$

$a(b-c)=ab-ac$

$a^2-b^2=(a+b)(a-b)$