Al Huda Science Academy Notes

10th Mathematics — New Book
Chapter 1: Complex Numbers

Review Exercise 1 Full Solution

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Board Exam Preparation Series

Q.5 — If $z_1 = 3 + 4i$ and $z_2 = 2 + 3i$, then verify:

(i) $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

$z_1 + z_2 = (3+4i) + (2+3i) = 5 + 7i$

L.H.S $= \overline{(z_1+z_2)} = \overline{(5+7i)} = 5 - 7i$

R.H.S $= \bar{z}_1 + \bar{z}_2 = (3-4i) + (2-3i) = 5 - 7i$

L.H.S = R.H.S = $5 - 7i$ ✓ Verified

(ii) $\overline{(z_1 z_2)} = \bar{z}_1 \cdot \bar{z}_2$

$z_1 z_2 = (3+4i)(2+3i) = 6 + 9i + 8i + 12i^2 = 6 + 17i - 12 = -6 + 17i$

L.H.S $= \overline{(z_1 z_2)} = \overline{(-6+17i)} = -6 - 17i$

R.H.S $= \bar{z}_1 \cdot \bar{z}_2 = (3-4i)(2-3i) = 6 - 9i - 8i + 12i^2 = 6 - 17i - 12 = -6 - 17i$

L.H.S = R.H.S = $-6 - 17i$ ✓ Verified

(iii) $\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$

L.H.S:

$z_1/z_2 = (3+4i)/(2+3i) \times (2-3i)/(2-3i) = [(3+4i)(2-3i)]/(4+9)$

$= (6 - 9i + 8i - 12i^2)/13 = (18 - i)/13 = 18/13 - (1/13)i$

L.H.S $= \overline{(z_1/z_2)} = 18/13 + (1/13)i$

R.H.S:

$\bar{z}_1/\bar{z}_2 = (3-4i)/(2-3i) \times (2+3i)/(2+3i) = [(3-4i)(2+3i)]/13$

$= (6 + 9i - 8i + 12)/13 = (18 + i)/13 = 18/13 + (1/13)i$

L.H.S = R.H.S = $18/13 + (1/13)i$ ✓ Verified

(iv) $|z_1| = |-z_1|$

$|z_1| = |3+4i| = \sqrt{(3^2+4^2)} = \sqrt{(9+16)} = \sqrt{25} = 5$

$-z_1 = -(3+4i) = -3-4i \Rightarrow |-z_1| = \sqrt{((-3)^2+(-4)^2)} = \sqrt{25} = 5$

$|z_1| = |-z_1| = 5$ ✓ Verified

(v) $\overline{(\bar{z}_2)} = z_2$

$z_2 = 2 + 3i \Rightarrow \bar{z}_2 = 2 - 3i \Rightarrow \overline{(\bar{z}_2)} = \overline{(2 - 3i)} = 2 + 3i$

$\overline{(\bar{z}_2)} = z_2$ ✓ Verified

(vi) $z_1\bar{z}_1 = |z_1|^2$

L.H.S $= z_1\bar{z}_1 = (3+4i)(3-4i) = 3^2 - (4i)^2 = 9 - 16i^2 = 9 - 16(-1) = 9+16 = 25$

R.H.S $= |z_1|^2 = (5)^2 = 25$

$z_1\bar{z}_1 = |z_1|^2 = 25$ ✓ Verified

Q.6 — If $z_1 = 5 + 4i$ and $z_2 = 3 + 2i$, then find:

(i) $z_1 z_2$

$z_1 z_2 = (5+4i)(3+2i) = 5\cdot3 + 5\cdot2i + 4i\cdot3 + 4i\cdot2i = 15 + 10i + 12i + 8i^2$

$= 15 + 22i - 8$

$z_1 z_2 = 7 + 22i$

(ii) z_1/z_2

$z_1/z_2 = (5+4i)/(3+2i) \times (3-2i)/(3-2i) = [(5+4i)(3-2i)]/(9+4)$

$= (15 - 10i + 12i - 8i^2)/13 = (15 + 2i + 8)/13 = (23+2i)/13$

$z_1/z_2 = 23/13 + (2/13)i$

(iii) $\bar{z}_1 \cdot \bar{z}_2$

$\bar{z}_1 \cdot \bar{z}_2 = (5-4i)(3-2i) = 5\cdot3 - 5\cdot2i - 4i\cdot3 + 4i\cdot2i = 15 - 10i - 12i + 8i^2$

$= 15 - 22i - 8$

$\bar{z}_1 \cdot \bar{z}_2 = 7 - 22i$

(iv) $|\bar{z}_1 \cdot \bar{z}_2|$

$|\bar{z}_1 \cdot \bar{z}_2| = |\bar{z}_1| \cdot |\bar{z}_2| = |z_1| \cdot |z_2|$

$= \sqrt{(5^2+4^2)} \times \sqrt{(3^2+2^2)} = \sqrt{(25+16)} \times \sqrt{(9+4)} = \sqrt{41} \times \sqrt{13}$

$|\bar{z}_1 \cdot \bar{z}_2| = \sqrt{533}$

Q.7 — Find the real and imaginary parts of $z = (2 + 7i)^{-1}$.

Multiply numerator and denominator by the conjugate (2 − 7i):

$z = 1/(2+7i) \times (2-7i)/(2-7i) = (2-7i)/[2^2-(7i)^2]$

$= (2-7i)/[4-49i^2] = (2-7i)/[4-49(-1)] = (2-7i)/(4+49) = (2-7i)/53$

Real Part = 2/53, Imaginary Part = −7/53

Q.8 — Solve the given simultaneous equations with complex coefficients for z and w:

$iz + (2 - i)w = 4 + i \quad ...(1)$

$iz + (3 + i)w = 3 + 3i \quad ...(2)$

Step 1 — Find w: Subtract (1) from (2):

$[(3+i) - (2-i)]w = (3+3i) - (4+i)$

$(1+2i)w = -1 + 2i$

$w = (-1+2i)/(1+2i)$

Multiply numerator and denominator by the conjugate (1−2i):

$w = [(-1+2i)(1-2i)] / [(1+2i)(1-2i)] = (-1 + 2i + 2i - 4i^2) / [1^2-(2i)^2]$

$= (-1 + 4i + 4) / [1-(-4)] = (3 + 4i)/5$

$w = 3/5 + (4/5)i$

Step 2 — Find z: Substitute w = 3/5 + (4/5)i into equation (1):

$iz + (2-i)(3/5 + (4/5)i) = 4 + i$

$iz + [2\cdot(3/5) + 2\cdot(4/5)i - i\cdot(3/5) - i\cdot(4/5)i] = 4 + i$

$iz + [6/5 + (8/5)i - (3/5)i - (4/5)i^2] = 4 + i$

$iz + [6/5 + (5/5)i - (4/5)(-1)] = 4 + i$

$iz + [6/5 + i + 4/5] = 4 + i \Rightarrow iz + (10/5 + i) = 4 + i$

$iz + (2 + i) = 4 + i \Rightarrow iz = 4 + i - 2 - i = 2$

$z = 2/i = [2\times(-i)] / [i\times(-i)] = -2i / 1$

$z = -2i$

Q.9 — Solve $(3 - 4i)(a + bi) = 1 + 0i$ and find the values of a and b.

Step 1: Expand the left side.

$(3-4i)(a+bi) = 3a + 3bi - 4ai - 4bi^2$

Since $i^2 = -1$: $= 3a + 3bi - 4ai + 4b = (3a + 4b) + (3b - 4a)i$

Given: $(3a + 4b) + (3b - 4a)i = 1 + 0i$

Step 2: Equate real parts.

$3a + 4b = 1 \quad ...(1)$

Step 3: Equate imaginary parts.

$3b - 4a = 0 \Rightarrow 3b = 4a \Rightarrow b = 4a/3 \quad ...(2)$

Step 4: Put value of b in (1).

$3a + 4(4a/3) = 1 \Rightarrow 3a + 16a/3 = 1 \Rightarrow (9a+16a)/3 = 1$

$25a/3 = 1 \Rightarrow 25a = 3 \Rightarrow a = 3/25$

Step 5: Find b.

$b = 4a/3 = (4/3) \times (3/25) = 4/25$

FINAL ANSWER: $a = 3/25$, $b = 4/25$

Q.10 — Solve the equation for x and y: $(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$

Step 1: Expand the LHS.

$(3-2i)(x+yi) = 3x + 3yi - 2xi - 2yi^2$

Since $i^2 = -1$: $= 3x + 3yi - 2xi + 2y = (3x + 2y) + (3y - 2x)i$

Step 2: Expand the RHS.

$2(x-2yi) + 2i - 1 = 2x - 4yi + 2i - 1 = (2x - 1) + (2 - 4y)i$

Step 3: Equate real parts.

$3x + 2y = 2x - 1 \Rightarrow x + 2y = -1 \quad ...(1)$

Step 4: Equate imaginary parts.

$3y - 2x = 2 - 4y \Rightarrow 7y - 2x = 2 \quad ...(2)$

Step 5: Solve the equations. From (1):

$x + 2y = -1 \Rightarrow x = -1 - 2y$

Put into (2):

$7y - 2(-1-2y) = 2 \Rightarrow 7y + 2 + 4y = 2 \Rightarrow 11y + 2 = 2 \Rightarrow 11y = 0 \Rightarrow y = 0$

Step 6: Find x.

$x = -1 - 2y = -1 - 2(0) = -1$

FINAL ANSWER: $x = -1$, $y = 0$

KEY FACTS — Review Exercise 1 Summary

Conjugate rule

Conjugate of $a + bi = a - bi$

Fundamental identity

$i^2 = -1$

Modulus identities

$z \times \bar{z} = |z|^2$

$|a + bi| = \sqrt{(a^2 + b^2)}$

Double conjugate

$\overline{(\bar{z})} = z$

Conjugate distributes over operations

$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

$\overline{(z_1 z_2)} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2 \quad (z_2 \neq 0)$

Tip

Always multiply by the conjugate of the denominator to perform division of complex numbers.