# Al-Huda Science Academy

## Mathematics

### Class 9

(Punjab Textbook Board — New Syllabus)

Covers all 13 chapters: DEFINITIONS, FORMULAS, KEY THEOREMS and SOLVED EXAMPLES

# Chapter 1: Real Numbers

## 1.1 Introduction

The set of real numbers, denoted by R, consists of all rational and irrational numbers. Every real number corresponds to a unique point on the number line, and every point on the number line corresponds to a real number.

## 1.2 Classification of Numbers

- Natural numbers (N): 1, 2, 3, 4, ...
- Whole numbers (W): 0, 1, 2, 3, ...
- Integers (Z): ..., -2, -1, 0, 1, 2, ...
- Rational numbers (Q): numbers of the form p/q where p, q are integers and $q \neq 0$
- Irrational numbers (Q'): numbers that cannot be written as p/q, e.g. $\sqrt{2}$, $\sqrt{3}$, $\pi$
- Real numbers (R) = Q ∪ Q'

## 1.3 Properties of Real Numbers

- Closure property: $a + b$ and $a \times b$ are real numbers for all real a, b
- Commutative property: $a + b = b + a$, $a \times b = b \times a$
- Associative property: $(a + b) + c = a + (b + c)$
- Distributive property: $a(b + c) = ab + ac$
- Existence of identity: $a + 0 = a$, $a \times 1 = a$
- Existence of inverse: $a + (-a) = 0$, $a \times (1/a) = 1$ $(a \neq 0)$

## 1.4 Properties of Equality and Inequality of Real Numbers

- Reflexive: $a = a$
- Symmetric: if $a = b$ then $b = a$
- Transitive: if $a = b$ and $b = c$ then $a = c$
- Trichotomy: for any two real numbers a and b, exactly one of $a < b$, $a = b$, $a > b$ holds

## 1.5 Radicals and Radicands

A radical is an expression of the form $\sqrt[n]{a}$, where n is called the index and a is the radicand.

$$\sqrt[n]{a} = a^{(1/n)}$$

Laws: $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ $\quad$ $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{a/b}$

## 1.6 Rationalization

Rationalizing the denominator means removing a radical from the denominator of a fraction by multiplying with a suitable conjugate.

### Example 1

**Question:** Rationalize $1/(\sqrt{5} - \sqrt{3})$.

Multiply numerator and denominator by the conjugate $(\sqrt{5} + \sqrt{3})$:

$1/(\sqrt{5}-\sqrt{3}) \times (\sqrt{5}+\sqrt{3})/(\sqrt{5}+\sqrt{3}) = (\sqrt{5}+\sqrt{3})/(5-3) = (\sqrt{5}+\sqrt{3})/2$

**Answer: $(\sqrt{5} + \sqrt{3})/2$**

# Chapter 2: Logarithms

## 2.1 Definition

If $a^x = y$ ($a > 0$, $a \neq 1$), then x is called the logarithm of y to the base a, written as x = log_a y.

$$a^x = y \iff \log_a y = x$$

## 2.2 Common and Natural Logarithms

- Common logarithm: base 10, written log y
- Natural logarithm: base e (e ≈ 2.71828), written ln y

## 2.3 Laws of Logarithms

$$\log_a(mn) = \log_a m + \log_a n \quad \text{(Product law)}$$
$$\log_a(m/n) = \log_a m - \log_a n \quad \text{(Quotient law)}$$
$$\log_a(m^n) = n \cdot \log_a m \quad \text{(Power law)}$$
$$\log_a a = 1, \ \log_a 1 = 0$$
$$\text{Change of base: } \log_a m = \text{log\_b } m / \text{log\_b } a$$

## 2.4 Characteristic and Mantissa

A common logarithm has two parts: the characteristic (integer part) and the mantissa (decimal part, always taken positive).

- For a number ≥ 1, characteristic = (number of digits before the decimal point) − 1
- For a number < 1, characteristic is negative and equals −(number of zeros right after the decimal point + 1)

## 2.5 Applications

Logarithms simplify calculations involving multiplication, division, powers and roots of large numbers, and are used in solving exponential equations.

### Example 1

**Question:** Simplify log 8 + log 5 using log 4 = 0.6021.

log 8 + log 5 = log(8 × 5) = log 40

log 40 = log(4 × 10) = log 4 + log 10 = 0.6021 + 1

**Answer: 1.6021**

# Chapter 3: Sets and Functions

### 3.1 Sets

A set is a well-defined collection of distinct objects, called elements or members.

- Sets are usually denoted by capital letters: A, B, C, ...
- Elements are denoted by small letters and membership is shown by $\in$ (belongs to) or $\notin$ (does not belong to)

### 3.2 Types of Sets

- Empty (null) set $\emptyset$: has no elements
- Singleton set: has exactly one element
- Finite set: has a countable number of elements
- Infinite set: has an unlimited number of elements
- Subset ($A \subseteq B$): every element of A is also in B
- Equal sets: $A = B$ if $A \subseteq B$ and $B \subseteq A$
- Universal set (U): the set containing all elements under consideration

### 3.3 Operations on Sets

*Union:* $A \cup B = \{x : x \in A \text{ or } x \in B\}$

*Intersection:* $A \cap B = \{x : x \in A \text{ and } x \in B\}$

*Difference:* $A - B = \{x : x \in A \text{ and } x \notin B\}$

*Complement:* $A' = \{x \in U : x \notin A\}$

### 3.4 Venn Diagrams

A Venn diagram represents sets as circles inside a rectangle (the universal set), used to visualize relationships and operations between sets.

### 3.5 Ordered Pairs and Relations

An ordered pair (a, b) has a as the first component and b as the second component, with $(a, b) \neq (b, a)$ unless $a = b$.

The Cartesian product $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

A relation from A to B is any subset of $A \times B$.

### 3.6 Functions

A function f from set A to set B is a special relation in which every element of A is associated with exactly one element of B.

- Domain: the set of all first elements (inputs)
- Range: the set of all second elements (outputs) actually used
- One-to-one function: distinct inputs give distinct outputs

- Onto function: every element of B is the image of some element of A

### Example 1

**Question:** If $A = \{1, 2, 3\}$ and $B = \{4, 5\}$, find $n(A \times B)$.

$n(A \times B) = n(A) \times n(B) = 3 \times 2$

**Answer: 6**

# Chapter 4: Factorization and Algebraic Manipulation

## 4.1 Important Algebraic Formulas

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

## 4.2 Factorization

Factorization means writing an algebraic expression as a product of two or more simpler expressions (factors).

- Common factor method: taking out the highest common factor
- Grouping method: grouping terms to find common factors
- Using algebraic identities listed above
- Factorizing quadratic trinomials of the form $ax^2 + bx + c$

## 4.3 H.C.F. and L.C.M. of Algebraic Expressions

The H.C.F. (highest common factor) is the product of all common factors, and the L.C.M. (least common multiple) is the product of all factors taken with their highest powers.

*H.C.F. × L.C.M. = product of the two expressions*

## 4.4 Algebraic Fractions

Simplification of algebraic fractions involves factorizing numerator and denominator and cancelling common factors.

### Example 1

**Question:** Factorize $x^2 - 9$.

$x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

**Answer: $(x + 3)(x - 3)$**

### Example 2

**Question:** Simplify $(x^2 - 4)/(x^2 + 4x + 4)$.

$x^2 - 4 = (x + 2)(x - 2)$

$x^2 + 4x + 4 = (x + 2)^2$

$(x+2)(x-2) / (x+2)^2 = (x-2)/(x+2)$

**Answer: $(x - 2)/(x + 2)$**

# Chapter 5: Linear Equations and Inequalities

### 5.1 Linear Equations in One Variable

A linear equation in one variable is of the form $ax + b = 0$ ($a \neq 0$), and has exactly one solution.

### 5.2 Linear Equations in Two Variables

An equation of the form $ax + by + c = 0$ (a, b not both zero) is a linear equation in two variables. Two such equations solved together form a system of linear equations.

### 5.3 Methods of Solving Simultaneous Linear Equations

- Elimination method: adding or subtracting equations to remove one variable
- Substitution method: expressing one variable in terms of the other and substituting
- Graphical method: plotting both lines; their intersection point is the solution
- Matrix (Cramer's rule) method for two equations in two unknowns

### 5.4 Linear Inequalities

An inequality compares two expressions using $<$, $>$, $\leq$ or $\geq$ instead of $=$.

- Adding or subtracting the same number from both sides keeps the inequality direction unchanged
- Multiplying or dividing both sides by a positive number keeps the direction unchanged
- Multiplying or dividing both sides by a negative number reverses the inequality sign

### 5.5 Graphing Linear Inequalities

The solution region of a linear inequality in two variables is the set of all points on one side of the boundary line, found by testing a point not on the line.

**Example 1**

**Question:** Solve the system: $x + y = 7$, $x - y = 1$.

Adding the equations: $2x = 8 \Longrightarrow x = 4$

Substitute in $x + y = 7$: $4 + y = 7 \Longrightarrow y = 3$

**Answer: $x = 4, y = 3$**

# Chapter 6: Trigonometry

## 6.1 Angles and Their Measurement

An angle is measured in degrees or radians.

$\pi$ *radians* $= 180°$ 1 *radian* $\approx 57.3°$

## 6.2 Trigonometric Ratios

For a right triangle with reference angle θ:

$\sin\theta = \text{opposite}/\text{hypotenuse}$ $\cos\theta = \text{adjacent}/\text{hypotenuse}$ $\tan\theta = \text{opposite}/\text{adjacent}$

$\csc\theta = 1/\sin\theta$ $\sec\theta = 1/\cos\theta$ $\cot\theta = 1/\tan\theta$

## 6.3 Values of Trigonometric Ratios for Special Angles

- 0°: sin = 0, cos = 1, tan = 0
- 30°: sin = 1/2, cos = √3/2, tan = 1/√3
- 45°: sin = 1/√2, cos = 1/√2, tan = 1
- 60°: sin = √3/2, cos = 1/2, tan = √3
- 90°: sin = 1, cos = 0, tan = undefined

## 6.4 Fundamental Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

## 6.5 Applications: Angle of Elevation and Depression

The angle of elevation is measured upward from the horizontal to a higher object; the angle of depression is measured downward from the horizontal to a lower object. Both are used to find heights and distances using trigonometric ratios.

### Example 1

**Question:** A ladder makes a 60° angle with the ground and reaches 10 m up a wall. Find the length of the ladder.

sin 60° = opposite/hypotenuse = 10/L

L = 10/sin 60° = 10/(√3/2) = 20/√3

**Answer: L ≈ 11.55 m**

# Chapter 7: Coordinate Geometry

## 7.1 The Cartesian Plane

The Cartesian plane is formed by two perpendicular number lines, the x-axis and y-axis, meeting at the origin O(0, 0). Any point is located by an ordered pair (x, y).

## 7.2 Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

## 7.3 Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

## 7.4 Slope (Gradient) of a Line

$$m = (y_2 - y_1)/(x_2 - x_1)$$

- Parallel lines have equal slopes
- Perpendicular lines have slopes whose product is −1

## 7.5 Equation of a Straight Line

*Slope-intercept form:* $y = mx + c$

*Point-slope form:* $y - y_1 = m(x - x_1)$

*Two-point form:* $(y - y_1)/(x - x_1) = (y_2 - y_1)/(x_2 - x_1)$

## 7.6 Area of a Triangle (Coordinate Form)

$$Area = \tfrac{1}{2} |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$$

**Example 1**

**Question:** Find the distance between A(2, 3) and B(6, 6).

$d = \sqrt{(6-2)^2 + (6-3)^2} = \sqrt{16 + 9} = \sqrt{25}$

**Answer: d = 5 units**

# Chapter 8: Logic

### 8.1 Statements (Propositions)

A statement is a sentence that is either true or false, but not both.

### 8.2 Logical Connectives

- Negation ($\neg p$): 'not p'
- Conjunction ($p \wedge q$): 'p and q' — true only when both are true
- Disjunction ($p \vee q$): 'p or q' — true when at least one is true
- Conditional ($p \rightarrow q$): 'if p then q'
- Biconditional ($p \leftrightarrow q$): 'p if and only if q'

### 8.3 Truth Tables

A truth table lists all possible truth values of a compound statement based on the truth values of its component statements.

### 8.4 Tautology, Contradiction and Contingency

- Tautology: a statement that is always true
- Contradiction: a statement that is always false
- Contingency: a statement that is sometimes true and sometimes false

### 8.5 Converse, Inverse and Contrapositive

For a conditional $p \rightarrow q$:

- Converse: $q \rightarrow p$
- Inverse: $\neg p \rightarrow \neg q$
- Contrapositive: $\neg q \rightarrow \neg p$ (always logically equivalent to the original statement)

# Chapter 9: Similar Figures (Ratio and Proportion in Geometry)

## 9.1 Ratio and Proportion

A ratio compares two quantities of the same kind. A proportion states that two ratios are equal: $a : b = c : d$.

## 9.2 Similar Triangles

Two triangles are similar if their corresponding angles are equal and their corresponding sides are in proportion.

- AA (Angle-Angle) similarity criterion
- SSS (side-side-side) similarity criterion
- SAS (side-angle-side) similarity criterion

## 9.3 Theorems on Proportion

- A line parallel to one side of a triangle divides the other two sides proportionally (Basic Proportionality / Thales' Theorem)
- The internal bisector of an angle of a triangle divides the opposite side in the ratio of the corresponding sides

## 9.4 Areas of Similar Figures

*Ratio of areas of similar figures = (ratio of corresponding sides)²*

**Example 1**

**Question:** The sides of two similar triangles are in the ratio 2:3. Find the ratio of their areas.

Ratio of areas = $(2/3)^2 = 4/9$

**Answer: 4 : 9**

# Chapter 10: Graphs of Functions

### 10.1 Graph of a Linear Function

The graph of $y = mx + c$ is a straight line with slope m and y-intercept c.

### 10.2 Graph of a Quadratic Function

The graph of $y = ax^2 + bx + c$ ($a \neq 0$) is a parabola, opening upward if $a > 0$ and downward if $a < 0$.

*Vertex x-coordinate:* $x = -b/2a$

### 10.3 Reading and Interpreting Graphs

- Intercepts: where the graph crosses the x-axis (roots) and y-axis
- Maximum/minimum point: the vertex of a parabola
- Increasing/decreasing intervals: where the graph rises or falls

### 10.4 Solving Equations Graphically

The real solutions of $f(x) = 0$ correspond to the x-coordinates where the graph of $y = f(x)$ crosses the x-axis.

# Chapter 11: Loci and Construction

### 11.1 Locus

A locus is the set of all points that satisfy a given geometric condition.

- The locus of points equidistant from two fixed points is the perpendicular bisector of the segment joining them
- The locus of points equidistant from the two sides of an angle is the bisector of that angle
- The locus of points at a fixed distance from a fixed point is a circle

### 11.2 Basic Geometric Constructions

- Bisecting a given line segment using compass and straightedge
- Bisecting a given angle
- Constructing a perpendicular to a line from a point
- Constructing an equilateral, isosceles or scalene triangle given specific measurements

### 11.3 Construction of Triangles

A triangle can be constructed when sufficient information is given, such as: three sides (SSS), two sides and the included angle (SAS), or two angles and a side (ASA).

# Chapter 12: Information Handling (Statistics)

## 12.1 Data and Its Types

- Raw data: data collected in its original, unorganized form
- Grouped data: data organized into class intervals with frequencies
- Frequency distribution table: shows how often each value or class occurs

## 12.2 Measures of Central Tendency

*Mean* $(\bar{x}) = \Sigma x / n$

*Median: the middle value when data is arranged in order*

*Mode: the value that occurs most frequently*

## 12.3 Measures of Dispersion

*Range = Highest value − Lowest value*

*Variance* $(\sigma^2) = \Sigma(x - \bar{x})^2 / n$

*Standard deviation* $(\sigma) = \sqrt{\text{Variance}}$

## 12.4 Graphical Representation of Data

- Bar graph: represents categorical data using rectangular bars
- Histogram: represents grouped numerical data with adjoining bars
- Frequency polygon: line graph joining midpoints of the tops of histogram bars
- Pie chart: represents data as sectors of a circle proportional to frequency

**Example 1**

**Question:** Find the mean of the data: 4, 8, 6, 5, 3, 10.

Sum = 4+8+6+5+3+10 = 36

Mean = 36/6

**Answer: Mean = 6**

# Chapter 13: Probability

### 13.1 Basic Terminology

- Experiment: an action with a well-defined set of possible results
- Sample space (S): the set of all possible outcomes
- Event: any subset of the sample space

### 13.2 Definition of Probability

$$P(E) = n(E) / n(S)$$

- $0 \leq P(E) \leq 1$
- P(sure event) = 1, P(impossible event) = 0

### 13.3 Laws of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

*For mutually exclusive events:* $P(A \cup B) = P(A) + P(B)$

$$P(A') = 1 - P(A)$$

### 13.4 Independent Events

Two events A and B are independent if the occurrence of one does not affect the probability of the other.

$$P(A \cap B) = P(A) \times P(B)$$

**Example 1**

**Question:** A fair die is rolled once. Find the probability of getting a number greater than 4.

S = {1,2,3,4,5,6}, n(S) = 6

Favorable outcomes = {5, 6}, n(E) = 2

P(E) = 2/6 = 1/3

**Answer: 1/3**